AN MINLP FORMULATION FOR SCHEDULING THE CLEANING OF HEAT EXCHANGER NETWORKS SUBJECT TO FOULING AND AGEING

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ABSTRACT

Fouling is an age-old problem in many process industries. It is responsible for large energy and throughput losses, resulting in financial penalties and negative environmental impact. One effective mitigation strategy is the regular cleaning of fouled heat transfer devices. Optimization tools can be used to schedule the timing of cleaning actions in order to minimize the cost of fouling and impact on productivity. In this work, the case where two cleaning methods are available is considered; therefore, the cleaning mode as well as the optimal cleaning intervals and the unit to be cleaned have to be determined. The two cleaning modes differ in their effectiveness in removing aged material. Ageing is assumed to transform the initial, soft, deposit, named ‘gel’, to a more cohesive material, named ‘coke’. An example would be a crude-oil preheat train where the operator has a choice between cleaning-in-place using solvents or mechanical cleaning wherein the unit has to be isolated for treatment. This aspect of choice has not been considered in previous studies of heat exchanger cleaning optimization. A mixed integer nonlinear programming (MINLP) problem is formulated for a heat exchanger network (HEN). Solutions are generated for the cases where ageing is quantified using a (i) simple two-layer model; (ii) temperature dependent two-layer model. The benefits of mixed-cleaning campaigns and the need for experimental data are highlighted.

INTRODUCTION

Fouling is recognized as the major unresolved problem in heat transfer processes. It is responsible for large energy and throughput losses, resulting in financial penalties and negative environmental impact.

One effective mitigation strategy is the regular cleaning of the fouled heat transfer units. As outlined by Wilson (2005), fouling and cleaning are symbiotic processes. However, along with choosing regular cleaning as a mitigation strategy for fouling, the challenge arises of deciding when to clean a unit in order to minimize energy losses. Furthermore, when dealing with a heat exchanger network (HEN) a second question needs to be answered; which unit is to be cleaned? This is an optimization problem where the cleaning actions and the selection of the unit must be scheduled in order to minimize energy losses and other associated operating costs.

In recent years, this problem has drawn the attention of the optimization community. The scheduling problem for a single HEX was considered earlier by Ma and Epstein (1981) and Casado (1990). Smalli et al. (1999) first considered the problem for a HEN. They applied a mixed-integer nonlinear programming (MINLP) model to an industrial network featuring eleven HEXs where the objective was to maximize the temperature of a specific stream over a relatively small time horizon, 120 days. This formulation was extended to larger networks in (Smalli et al., 2001) where the objective was to minimize the total operating cost over a three year horizon. Georgiadi and coworkers (2000a,b) crafted a different MINLP scheduling model which they were able to transform to a mixed-integer linear programming (MILP) problem which is much easier to solve. They replaced the logarithmic temperature difference (LMTD) term in the heat transfer equation:

\[ Q = UA(LMTD) \]  

where \( Q \) is the heat duty, \( U \) is the overall heat transfer coefficient and \( A \) the heat transfer area, with the arithmetic mean value.

All the above groups reported that the MINLP model describing the process is highly non-convex. Thus, one cannot guarantee the global optimality of the obtained solution. Moreover, the combinatorial nature of the formulated problem, deriving from the use of integer variables, poses a constraint on the size of the model.

Lavaja and Bagajewicz (2004) proposed a rigorous MILP formulation to overcome the difficulties associated with the non-convex MINLP models. More importantly, they achieved this without introducing any linear approximations to the nonlinear equations related to heat transfer or fouling models. After algebraic manipulations, the nonlinear equations contain only products of integer variables which can be readily linearized by adding new continuous variables and supplementary constraints. The importance of this accomplishment lies in the fact that MILP models can be solved to yield globally optimal solutions. Nonetheless, transforming the model comes with a price, namely the size of the model. As stated by Lavaja and Bagajewicz, the resulting MILP formulation is...
extremely time-consuming when solved for whole networks, and so they proposed a decomposition method to speed-up the solution procedure. The results obtained for a fixed network using this decomposition technique were demonstrated to be superior than those obtained using MINLP models.

These different approaches demonstrated the potential benefits of using optimization tools rather than heuristic rules to develop cleaning schedules. It is obvious, by performing cleaning actions according to such schedules, even if these are sub-optimal solutions, the energy losses and the operating cost are significantly lower (Lavaja and Bagajewicz, 2004). The approaches must, however, be able to incorporate more detailed fouling models, if additional gains are to realized and describe real behaviours.

In this work, the impacts of ageing on fouling and cleaning dynamics are considered. Ageing is the transformation of the initial soft deposit into a more cohesive form over time, due to extended exposure to conditions at the heat exchanger wall. Epstein (1983) identified ageing as one of the primary mechanistic steps in the process of fouling. A detailed description of this phenomenon and its impact on heat transfer for the case of chemical reaction fouling was considered by Ishiyama et al. (2011b).

The extent of ageing determines the state of the deposit and therefore the ease with which the layer can be removed. In practice, an operator must make a choice between different cleaning methods (see Müller-Steinhagen, 2000), and the primary factor considered by an operator will be whether it can remove the deposit. However, a cleaning method may be used even if it is not 100% effective if it can be performed quickly. This element of choice is the subject of this work and revisits the scheduling problem for a HEN when two cleaning methods are available. The first technique, typically a cleaning-in place (CIP) one, can only remove fresh deposit while the second, ex-situ, method removes the harder aged layer as well. Performing a number of cleanings using the CIP method can extend the operating time between ex-situ cleans considerably.

The scheduling problem was introduced by Ishiyama et al. (2011b) and Pogiatzis et al. (2011). The former employed heuristic techniques to identify cleaning schedules while the latter compared those heuristics with simple optimization approaches. The use of mixed cleaning strategies (combinations of CIP and ex-situ techniques) yields a cleaning super-cycle, where the ex-situ method resets the unit to the clean state and the sequence can be repeated.

A detailed description of the optimization problem for a single fouled HEX which accounts for the effects of ageing on heat transfer and the choice between the two cleaning methods will be presented in Pogiatzis et al. (in preparation). This formulation of the scheduling model is adapted here for a network and a brief description is given in the section that follows.

PROBLEM FORMULATION

To formulate the optimization problem, it is, at first, essential to establish the connections between fouling - ageing kinetics, the mathematical model describing the operation of the HEXs and the cleaning dynamics. After doing so, the appropriate objective function needs to be introduced along with the some necessary constraints for the network and the discretization scheme for time.

Fouling Analysis

The key requirement, for the purposes of obtaining the optimal cleaning schedule, is to be able to track the effect of fouling on heat recovery and on cleaning effectiveness. Thus, the two-layer model presented by Ishiyama et al. (2011a) is used to quantify the energy losses associated with fouling and to impose a criterion on the selection of the appropriate cleaning method. Figure 1 illustrates this two-layer concept.

Treating the two layers as a pair of thin insulating slabs, the total thermal fouling resistance, $R_f$, is calculated as the sum of the thermal resistances of each layer, viz.

$$R_f = R_{f,g} + R_{f,c} = \frac{\delta_g}{\lambda_g} + \frac{\delta_c}{\lambda_c}$$  \hspace{1cm} (2)

where $\lambda_g$ and $\lambda_c$ are the thermal conductivities of the gel and coke layers, respectively. The zero order growth rate of the gel layer is given by

$$\frac{d\delta_g}{dt} = \frac{dR_{f,g}}{dt} = \lambda_g - \frac{dR_{f,c}}{dt} \lambda_c = k_g \lambda_g - k_c \lambda_c$$  \hspace{1cm} (3)

and for the coke layer by

$$\frac{d\delta_c}{dt} = \begin{cases} \frac{dR_{f,c}}{dt} \lambda_c & \text{if } \delta_g > 0 \\ 0 & \text{if } \delta_g = 0 \end{cases}$$  \hspace{1cm} (4)

where $k_g$ and $k_c$ are the deposition and ageing rates, respectively. These rates are assumed to follow an Arrhenius dependency on temperature:

Figure 1: Schematic of a fouling deposit undergoing ageing on a heat transfer wall. Darker shading indicates harder material: $\delta_g$ and $\delta_c$ are the thicknesses of the gel and coke layers, respectively.
where $a_g$ and $a_c$ are the pre-exponential factors for the two processes and $E_g$ and $E_c$ their respective activation energies. The temperature at the gel surface, $T_g$, and the temperature at the coke-gel interface, $T_c$, are shown in Figure 2.

The temperature distribution concept presented here is based on the one described by Ishiyama et al. (2009).

It is postulated that deposition and ageing occur at a uniform rate across the tubes. The temperature distribution is calculated at both ends of the HEX, giving two growth rates for each layer. The arithmetic mean of these rates is used in the calculations.

**Scheduling Analysis**

First of all, it must be established what information the optimal solution should include. As mentioned before, this is: (i) the time instants of the cleanings; (ii) which unit to clean at each instant and (iii) how to clean the chosen unit.

Clearly, this scheduling problem is combinatorial in nature. Furthermore, fouling kinetics and heat transfer process are described by nonlinear equations, yielding a MINLP problem. It is worth emphasizing, once again, that such problems are very hard to solve to optimality especially if they are non-convex. The scheduling problem examined here is highly non-convex and therefore, it is very computationally expensive to obtain even locally optimal solutions.

**Objective function.** The scheduling problem for a network involves the minimization of the total operating cost (TOC) due to fouling over a fixed operating horizon, $\tau$. Thus the objective function takes the form:

$$TOC \equiv \text{energy losses} + \text{cleaning cost}$$

$$TOC = \left\{ \sum_{n=1}^{N_u} f_e \int_0^{T_{op}} (Q_{ci} - Q(t)) \, dt \right\} + \sum_{i=1}^{N_{mc}} C_{mc} + \sum_{j=1}^{N_{chc}} C_{chc}$$

where $N_u$ is the number of units, $f_e$ is the cost of energy, $Q_{ci}$ is the heat duty of the clean HEX, $Q(t)$ is the heat duty during the operating period and $N_{mc}$ and $N_{chc}$ are the number of mechanical and chemical cleanings performed. Finally, $C_{mc}$ and $C_{chc}$ refer to the cost of mechanical and chemical actions.

The integral term takes into account the energy losses due to fouling and cleaning actions. Also, the cost of the cleaning actions is added.

**Time representation.** The MINLP problem formulated here is also of dynamic nature because of equations (3), (4) and (8). Hence, an integration scheme is required to obtain numerical solutions for the integral and differential equations. For that purpose, orthogonal collocation is chosen for its precision and requirement of relatively fewer discretization points, thus resulting in a transcription of the problem into a smaller size NLP. Equations (3), (4) and (8) are approximated using Lagrange polynomials of 5th order and the collocation points are chosen to be Radau’s roots (Biegler, 2010).

Time must be discretized for collocation to be applied. Firstly, the total operating time is divided into a number of periods, $N_p$, and then each period is divided into three elements. The first and second elements correspond to the durations of chemical, $\Delta t_{chc}$, and mechanical, $\Delta t_{mc}$, cleanings, respectively. The third element, considered to be the operating sub-period, has length $\Delta t_{op}$. The sum of the durations of these three elements is thirty days. Within each element six nodes are placed. Figure 3 shows a graphic representation of a discrete period.
The total operating time, \( \tau \), is:

\[ \tau = N_p (\Delta t_{chc} + \Delta t_{mc} + \Delta t_{op}) \]  

(9)

**Constraints.** There are some necessary constraints that need to be imposed in the problem. The selection of the unit and cleaning technique is represented by a set of binary variables, \( y_{u,l} \), where the subscript ‘u’ denotes the unit and the subscript ‘l’ the cleaning technique. The first constraint ensures that each unit is either cleaned using one and only one of the available methods or is not cleaned, viz.

\[ y_{u,chc} + y_{u,mc} \leq 1 \]  

(10)

where, as above, ‘chc’ denotes chemical cleaning and ‘mc’ mechanical cleaning. The next constraint refers to blocks of parallel units (only two units in this work). Here, only one unit can be off-line at a time instant:

\[ \sum_{pu} \sum_{l=chc,mc} y_{u,l} \leq 1 \]  

(11)

where the subscript ‘pu’ stands for parallel units.

Lastly, for blocks of parallel units the feeds, cold and hot, are split equally between the devices. When a unit of the block is cleaned the whole feed goes through the other heat exchanger.

**Solution Approaches**

The scheduling model is coded in GAMS™ (Brooke et al., 2005) and solved using DICOPT© on an ASUS Chassis: AMD Athlon Processor 2.21 GHz PC. In DICOPT©, the MINLP problems are solved by implementing an outer approximation/equality relaxation (OA/ER) algorithm (Kocis and Grossmann, 1987).

Unfortunately, for the one year time-horizon considered here, DICOPT© converged only for the case where the deposit and ageing rates were fixed to a certain value. When these rates are functions of temperature, as discussed above, the model becomes very nonlinear. As a result, the optimizer fails to converge. To overcome this set-back a simple stochastic method is employed. The binary variables are fixed in random feasible combinations and the model is simulated. At the end, the combination which gives the lowest value of the objective function is chosen as the best-obtained solution.

**RESULTS**

Results are presented for three case studies. Case I is for constant deposit and ageing rates. Case II and III involve temperature dependent rates. In case III the ageing rate is faster.

A schematic representation of the network examined is shown in Figure 4. The parameters for the heat exchanger, cleaning analysis and fouling rates are given in Tables 1-3.

![Figure 4: Configuration of network. Solid lines denote cold streams; dashed lines, hot streams. \( T_{ex} \) is the exit temperature of the cold stream.](image)

**Table 1: HEX performance parameters.**

<table>
<thead>
<tr>
<th>HEX</th>
<th>( T_{c,in} ) (K)</th>
<th>( T_{h,in} ) (K)</th>
<th>( U_{cl} ) (kW/m(^2) K)</th>
<th>( A ) (m(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1A</td>
<td>300</td>
<td>590</td>
<td>0.34</td>
<td>96</td>
</tr>
<tr>
<td>1B</td>
<td>300</td>
<td>590</td>
<td>0.34</td>
<td>72</td>
</tr>
<tr>
<td>2A</td>
<td>-</td>
<td>650</td>
<td>0.34</td>
<td>239</td>
</tr>
<tr>
<td>2B</td>
<td>-</td>
<td>650</td>
<td>0.54</td>
<td>143</td>
</tr>
</tbody>
</table>

**Table 2: Cleaning parameters.**

<table>
<thead>
<tr>
<th>Cleaning</th>
<th>Chemical</th>
<th>Mechanical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>£ 2000</td>
<td>£ 6000</td>
</tr>
<tr>
<td>Duration</td>
<td>1 day</td>
<td>5 days</td>
</tr>
</tbody>
</table>

**Table 3: Arrhenius model parameters for deposition (Equation (5)) and ageing (Equation (6)).**

<table>
<thead>
<tr>
<th>Case</th>
<th>( E_d ) (kJ/mol)</th>
<th>( a_d ) (m(^2) K/kW s)</th>
<th>( E_c ) (kJ/mol)</th>
<th>( a_c ) (m(^2) K/kW s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0</td>
<td>5.8\times10^{-11}</td>
<td>0</td>
<td>2.9\times10^{-12}</td>
</tr>
<tr>
<td>II</td>
<td>62</td>
<td>8.1\times10^{-9}</td>
<td>80</td>
<td>1.16\times10^{-4}</td>
</tr>
<tr>
<td>III</td>
<td>62</td>
<td>8.1\times10^{-9}</td>
<td>77</td>
<td>1.39\times10^{-4}</td>
</tr>
</tbody>
</table>

The cold stream mass flow rate is 135 kg/s and its thermal capacity is 3125 kJ/kg K. The hot stream thermal capacity is 2200 kJ/kg K and the mass flow rates are 128 kg/s and 115 kg/s for hot stream A and B, respectively. The thermal conductivity of the gel layer is 0.1\times10^{-3} kW/m K and of the coke layer 1\times10^{-3} kW/m K. The energy cost, \( f_e \), is 0.5 £/kW.
Figure 5: (a) Cleaning schedules for case studies I-III. Circles denote chemical cleaning actions and triangles mechanical cleaning at the beginning of the period. $N_{ca}$ is the total number of cleaning actions. (b) Final temperature of cold stream for case studies I-III. Solid line, best-obtained schedule; dotted line, no cleaning.
The best-obtained cleaning schedule for each scenario is presented in Figure 5a, while Figure 5b shows the corresponding exit temperature of the cold stream alongside that obtained without any cleaning.

The results are summarized in Table 4. The different operating costs for Cases II and III when no cleaning is performed are due to the difference in ageing rates. In Case III the deposition rate is the same as in Case II, but the ageing rate is faster. Thus, the growth of the coke layer is faster while the opposite goes for the gel layer. Consequently, the energy losses in Case III are smaller since the coke layer is more conductive than the gel layer.

Table 4: Operating cost of each scenario for no cleaning and for the best cleaning schedule.

<table>
<thead>
<tr>
<th>Case</th>
<th>No cleaning</th>
<th>Best cleaning schedule</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N_c</td>
<td>N_chc</td>
</tr>
<tr>
<td>I</td>
<td>897 k£</td>
<td>20</td>
</tr>
<tr>
<td>II</td>
<td>3443 k£</td>
<td>17</td>
</tr>
<tr>
<td>III</td>
<td>1828 k£</td>
<td>14</td>
</tr>
</tbody>
</table>

For Case I, the MINLP model was solved for 20 different starting points. The cleaning schedule which gives the smallest operating cost, of 396 k£, includes 19 chemical cleanings and one mechanical cleaning. As seen in Figure 5b (I), the drop of in cold stream exit temperature caused by fouling is moderate and relatively linear.

For Case II, a random sampling search was performed for 410 different combinations of the binary variables. The best-obtained solution yielded an operating cost of 1272 k£ which is 63% lower than the operating cost when the units are not cleaned. The drop in the final temperature for this case is rapid due to the fast growth of the gel layer with its low thermal conductivity. As shown in Figure 5b (II), when no cleaning is performed the exit temperature reduces by 80 K in one year. The operating cost can be reduced further if the constraint for equal flow splits between parallel units is removed. This will yield a NLP problem when the binary variables are fixed. Solving this NLP problem for the best-obtained solution of Case II gives a TOC of 1231 k£. The model in Case III was solved for 50 random samples and the best-obtained schedule is presented in Figure 5a (III). It includes 14 cleaning actions, of which 8 are mechanical. The faster ageing rate, compared to Case II, explains the regular mechanical cleaning of the units. Removing the flow split constraint and optimizing the split profile reduces the operating cost by a further 14 k£.

**DISCUSSION**

The MINLP model for scheduling the cleaning actions on a heat exchanger network undergoing fouling and ageing was successfully solved for the case study where the deposit and ageing rates were constant. It failed to converge in the other two case studies where the fouling rates depended on the temperatures of the two layers. This was due to the highly nonlinear coupled equality constraints introduced to the model by the Arrhenius temperature dependency. The time horizon explored, hence the size of the model, also affected the convergence of the optimizer.

Furthermore, even for case I where the optimizer was able to produce a solution, it is observed that different starting points yield different locally optimal solutions. The non-convex character of the model is responsible for the large number of sub-optimal solutions that exist.

However, even the non-optimal solutions obtained for cases II and III lead to large energy and money savings. Also, they provide a useful insight for the problem. It is obvious, by comparing the results for the two scenarios, that ageing affects strongly the choice and the timing of the cleaning mode. The explanation lies on the different thermal conductivities of the two layers, which plays an important role in the whole process especially when carbonaceous materials are involved (Ishiyama et al., 2011b). Because of the hardened material’s higher thermal conductivity the thermal effect of fouling is decreased. In addition, the effectiveness of some cleaning methods is limited by ageing.

Lastly, it is shown that the optimization of the splits between parallel units can lead to additional energy savings. This is an important aspect of the mitigation strategy and the scheduling problem and it must be considered at all times.

**CONCLUSIONS**

1. The optimization problem of scheduling the cleaning actions for a heat exchanger network subject to fouling was revisited. The new MINLP formulation takes into account the effects of ageing on fouling and cleaning dynamics. Furthermore, an extra decision variable is added to the scheduling model; the decision between two cleaning methods which differ in their effectiveness to remove the aged material.

2. The MINLP model was successfully solved by a commercial optimizer when the growth rates were described by zero order kinetics, but, failed to converge when the rates were functions of the temperature. This model is highly nonlinear and non-convex and in addition to its combinatorial nature it is concluded that obtaining a global solution is a very difficult task.

3. A random sampling search was used to obtain a solution for the scenarios where an Arrhenius dependency on temperature described the growth of the layers. Despite the non-optimal nature of the best-obtained solutions the benefits of mixed cleaning campaigns are evident for the combinations of deposit-ageing rates chosen.

4. The importance of reliable estimations for the fouling model parameters is also underlined. It is expected that the optimal schedules will be highly sensitive to these parameters. Therefore, experimental data are needed for the construction of detailed and realistic fouling models.

5. The scope of ongoing and future work is to overcome the drawbacks associated with the MINLP formulation. Alternative model structures and different deterministic search techniques are to be explored.
ACKNOWLEDGMENTS

Funding for Thomas Pogiatzis from Onassis Foundation is gratefully acknowledged. A travel grant for TP from Homerton College, University of Cambridge, is also acknowledged.

NOMENCLATURE

Latin

- $A$: heat transfer area, m$^2$
- $a_c$: Arrhenius constant for coke, m$^2$ K/kW s
- $a_g$: Arrhenius constant for gel, m$^2$ K/kW s
- $C_{mc}$: cost of mechanical cleaning, £
- $C_{chem}$: cost of chemical cleaning, £
- $d_{int}$: tube internal diameter, m
- $E_c$: activation energy for coke, kJ/mol
- $E_g$: activation energy for gel, kJ/mol
- $f_e$: energy cost, £/kW
- $h_{co}$: cold stream heat transfer coefficient, kW/m$^2$ K
- $h_h$: hot stream heat transfer coefficient, kW/m$^2$ K
- $k_c$: thermal resistance rate of coke, m$^2$ K/kW s
- $k_g$: thermal resistance rate of gel, m$^2$ K/kW s
- $\Delta T$: log mean temperature difference, K
- $N$: number
- $Q$: heat duty, kW
- $R$: gas constant, kJ/mol K
- $R_c$: thermal resistance of coke, m$^2$ K/kW
- $R_g$: thermal resistance of gel, m$^2$ K/kW
- $R_f$: fouling thermal resistance, m$^2$ K/kW
- $t$: time, s
- $T$: temperature, K
- $T_{ex}$: exit temperature of cold stream, K
- $T_i$: gel-coke interface temperature, K
- $T_s$: gel surface temperature, K
- $TOC$: total operating cost, £
- $U$: heat transfer coefficient, kW/m$^2$ K
- $y$: binary variables

Greek

- $\delta_c$: thickness of coke, m
- $\delta_g$: thickness of gel, m
- $\Delta t$: time interval, s
- $\lambda_c$: thermal conductivity of coke, kW/m K
- $\lambda_g$: thermal conductivity of gel, kW/m K
- $\tau$: operating period, s

Subscript

- $c$: coke
- $ca$: cleaning actions
- $ chc$: chemical cleaning
- $cl$: clean
- $co$: cold
- $c$: coke
- $ex$: exit
- $f$: fouling
- $g$: gel
- $h$: hot
- $i$: interface
- $mc$: mechanical cleaning
- $p$: period
- $pu$: parallel units
- $s$: surface
- $u$: unit

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