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ANALYSIS OF HIGHLY NOISY CRUDE OIL FOULING DATA USING KALMAN FILTER

A. Mirsadraee and M.R. Malayeri

Institute for Thermodynamics and Thermal Engineering (ITW), University of Stuttgart,
Pfaffenwaldering 6, D-70550, Stuttgart, Germany (m.malayeri@itw.uni-stuttgart.de)

ABSTRACT

The present study uses Kalman filter as a robust mathematical tool to accurately and reliably estimate crude oil fouling resistances of HTRI data banks. HTRI conducted a comprehensive set of crude oil fouling runs which will be used as source data in this study. Noise, a coherent feature of many fouling processes which can result from process irregularities, are considered in the model. The Kalman filter minimizes the estimation error covariance by considering the measurement and process noise covariance matrices. This, in turn, requires that process and measurement noise covariance are carefully chosen by the statistical analysis of measurements and estimations. The relation between these two defines the smoothness and shape of the estimated trend of fouling resistance. The comparison of the experimental data and model confirmed the reliability of the applied method for qualitative and quantitative estimation of fouling resistance for various operating conditions of crude oil fouling.

INTRODUCTION

Oil refineries nowadays have to cope with heavier crude oils or densified residuals with higher risk of fouling from sources which until recently have not been economical to process. Mechanisms of crude oil fouling in turn are complicated and include many parameters with poorly understood interactions. An accurate and reliable estimation of fouling propensity is based on measurements from the experimental studies or field operation. The experimental results can, nonetheless, be noisy or sparse to some extent for various reasons. For instance, faulty sampling of temperature sensors during the experiments, or formation of fouling layer on the temperature probe which results in fluctuations of different magnitudes. Other examples include changes in the feed composition or operating conditions such as crude velocity or temperature.

One viable option to deal with such problems is to use non-parametric tools i.e. neural networks, Kalman filters which have the capability of learning complex processes much faster and more accurately. They are based on continuously updated exchanger functioning modes and detective monitoring tools of when the deposition would occur. In the past few years, in particular, there have been further advancements in approximation tools that have facilitated the interpretation of highly non-linear and time-dependent data with more reliability.

The Kalman filter is an *optimal recursive data processing algorithm* [1, 2, 3]. It incorporates process parameters regardless of their uncertainties to estimate the current state of interest i.e. fouling resistance. The possible information which can be fed to the model are the mathematical process and measurement models, the uncertainties of the mathematical process and measurement model, and the initial conditions of the variables of interest, i.e. fouling resistance and its first and second order gradients with respect to time. The advantages of the Kalman filter over other filtering methods are that it:

- uses both mathematical process model and measurement to estimate the fouling resistance (state of the system) where other methods just use the measurements and estimates the process states.
- is able to consider the uncertainties of the mathematical process model and measurements in its estimations.
- predicts of fouling trend for multi-step ahead. See the additional text

The Kalman filter can be employed for estimating the crude oil fouling for a wide range of operating conditions under various scenarios regarding to the characteristics of the fouling resistance data. These include experimental data which are noisy, sparse, with short or long fouling time exposures which may follow a linear or asymptotic trend with or without fouling induction period. The Kalman filter provides an option to estimate the fouling resistance of crude oil preheat trains under the mentioned conditions.

THE KALMAN FILTER

Consider a nonlinear discrete-time system as represented in Eq. (1). A time discrete equation is a formula for computing the process state (e.g. fouling resistance) at any time step which is based on its values at previous time step. The vector of process states, $\mathbf{x} \in \Re^n$, can in fact be any of dependent variables of the process and their derivations as:

$$\mathbf{x}_{k+1} = \mathbf{A}_k \mathbf{x}_k + \mathbf{B}_k \mathbf{u}_k + \boldsymbol{\omega}_k \tag{1}$$

where $\mathbf{x}_k$ is a $n \times 1$ system state matrix at t_k. Here n is the number of states at t_k. i.e. R_f ,$\dot{R}_f$,$\ddot{R}_f$ and $\mathbf{A}_k$ is the $n \times n$ matrix of state transition at time t_k, a matrix whose product with the state matrix at previous time step gives the state value at the latter time steps. $\mathbf{B}_k$ is a $n \times l$ input transition matrix at time t_k, and $\mathbf{u}_k$ is an optional input matrix of $l \times 1$ which includes the operating conditions of the heat exchanger and could be zero, constant or variable with respect to t_{k-1}. In Eq. (1), $\boldsymbol{\omega}_k$ is a n-dimensional vector of the process noise with a process noise covariance of $\mathbf{Q}_k$ at t_k. The initial process states, $\mathbf{x}_0$, is a vector with a known mean value for each of its elements. The expected value of a variable is usually its mean value at that time step, $\mathbb{E}[\mathbf{x}_0] = \boldsymbol{\mu}_0$ with the covariance $\mathbf{P}_0 = \mathbb{E}[(\mathbf{x}_0 - \boldsymbol{\mu}_0)(\mathbf{x}_0 - \boldsymbol{\mu}_0)^T]$. The $\mathbb{E}$ sign denotes the expected value of the process state vector, $\mathbf{x}_k$.

The measurement vector, $\mathbf{y} \in \Re^m$, can be written as:

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \boldsymbol{\vartheta}_k \tag{2}$$

where $\mathbf{H}_k$ is a $m \times n$ matrix of ideal connection between the measurement and the state matrix at t_k (m is the number of measurement data points) and $\boldsymbol{\vartheta}_k$ is a m-dimensional vector of measurement noise with measurement noise covariance of $\mathbf{R}_k$ at t_k. A direct connection matrix between the measurement and state matrices is made through a conversion matrix, $\mathbf{H}_k$, whenever no additional gain or offset is needed, it is then called the ideal connection matrix.

The vector of process states, $\mathbf{x}_k = (x_{1k}, \dots, x_{nk})$ is n-dimensional and in this study, the vector of process state, is including of the fouling resistance, its first and second order gradients with respect to time. However, the first derivation of fouling resistance has not to be misinterpreted with the *initial fouling rate* that can be estimated by many available correlations like as Ebert and Panchal [4, 5] or the modified one by Polley et al. [6]. The vector of measurements, $\mathbf{y}_k = (y_{1k}, \dots, y_{mk})$ is m-dimensional and includes fouling resistances. The process and measurement noise vectors, $\boldsymbol{\omega}_k$ and $\boldsymbol{\vartheta}_k$, are white with a Gaussian distribution and have known or unknown covariances.

The process and measurement noise vectors are assumed to be independent of each other. The white noise is a random signal with a flat (constant) power spectral density. The value of such noise can be estimated using a probability function with a constant or variable value for a set of samples thus:

$$\mathbb{E}[\boldsymbol{\omega}_k] = 0 \qquad \begin{array}{ll} \mathbb{E}\left[\boldsymbol{\omega}_k \boldsymbol{\omega}_j^T\right] = 0 & \text{for } k \neq j \\ \mathbb{E}\left[\boldsymbol{\omega}_k \boldsymbol{\omega}_j^T\right] = \mathbf{Q}_k & \text{for } k = j \\ \mathbb{E}[\boldsymbol{\omega}_k \mathbf{x}_0^T] = 0 \text{ for all k} & \end{array} \tag{3}$$

$$\mathbb{E}[\boldsymbol{\vartheta}_k] = 0 \qquad \begin{array}{ll} \mathbb{E}\left[\boldsymbol{\vartheta}_k \boldsymbol{\vartheta}_j^T\right] = 0 & \text{for } k \neq j \\ \mathbb{E}\left[\boldsymbol{\vartheta}_k \boldsymbol{\vartheta}_j^T\right] = \mathbf{R}_k & \text{for } k = j \\ \mathbb{E}[\boldsymbol{\vartheta}_k \mathbf{x}_0^T] = 0 \text{ for all k} & \end{array} \tag{4}$$

A priori estimate gives the knowledge of the process states ($\mathbf{x}_k$) prior to step k and can be denoted as $\hat{\mathbf{x}}_k^- \in \Re^n$, and $\hat{\mathbf{x}}_k^+ \in \Re^n$ is the *a posteriori* state estimation at step k which includes the effect of measurement, $\mathbf{y}_k$ (consider the hat symbol (^) for estimations, the minus sign (−) for *a priori* estimations, and the positive sign (+) for *a posteriori* estimations). Thus, *a priori* and *a posteriori* errors of estimation can be defined as:

$$\mathbf{e}_k^- = \mathbf{x}_k - \hat{\mathbf{x}}_k^- \tag{5}$$

$$\mathbf{e}_k^+ = \mathbf{x}_k - \hat{\mathbf{x}}_k^+ \tag{6}$$

Consequently, the *a priori* and *a posteriori* estimation error covariance are:

$$\mathbf{P}_k^- = \mathbb{E}\left[\mathbf{e}_k^- \mathbf{e}_k^{-T}\right] \tag{7}$$

$$\mathbf{P}_k^+ = \mathbb{E}\left[\mathbf{e}_k^+ \mathbf{e}_k^{+T}\right] \tag{8}$$

The *a posteriori* state estimation $\hat{\mathbf{x}}_k^+$ can be calculated as a linear combination of an *a priori* estimation $\hat{\mathbf{x}}_k^-$ and a weightened difference between an actual measurement $\mathbf{y}_k$ and measurement which may be calculated by *a priori* estimations, $\mathbf{H}_k \hat{\mathbf{x}}_k^-$ as:

$$\hat{\mathbf{x}}_k^+ = \hat{\mathbf{x}}_k^- + \mathbf{K}_k(\mathbf{y}_k - \mathbf{H}_k \hat{\mathbf{x}}_k^-) \tag{9}$$

The difference $(\mathbf{y}_k - \mathbf{H}_k \hat{\mathbf{x}}_k^-)$ is called the *measurement innovation* or *residual*. It principally shows the difference between the predicted measurements, $\mathbf{H}_k \hat{\mathbf{x}}_k^-$ and actual measurements, $\mathbf{y}_k$.The blending factor in Eq. (9), $\mathbf{K}_k$, which minimizes the *a posteriori* estimation error covariance is called the Kalman gain. By substitution of Eq. (9) into Eq. (6) and then setting the derivation of the $\mathbf{P}_k^+$ with respect to $\mathbf{K}_k$ equal to zero and then solving for $\mathbf{K}_k$, the Kalman gain, can then be obtained in following form:

$$\mathbf{K}_k = \frac{\mathbf{P}_k^- \mathbf{H}_k^T}{\mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^T + \mathbf{R}_k} \tag{10}$$

Once the Kalman gain has minimized the *a posteriori* estimation error covariance then it can be utilized for updating the *a priori* estimation of the process states, $\hat{\mathbf{x}}_k^-$. In order to implement the Kalman filter, a series of equations has to be calculated.

The initial guess for the process states ($\hat{\mathbf{x}}_0^+$) and the *a posteriori* estimation error of process states ($\mathbf{P}_0^+$) at $t = 0$ is needed, then the recursive algorithm of Kalman filter for estimating process states proceeds as follows:

1) <u>Time update step</u> (projecting the state and covariance estimations forwards from last time step)

1.1) Calculating the *a priori* state estimation using the mathematical process model.
1.2) The *a priori* estimation leads to the computation of *a priori* estimation error covariance.

2) Measurement update step (correcting the predictions made in the current time step).
2.1) Calculation of the Kalman gain.
2.2) Calculation of the *a posteriori* state estimation and *a posteriori error* covariance.
2.3) The results of the previous step (2.2) are then fed to step (1.1) for estimating the next state of the process (time update step).

The *a posteriori* estimation of process states is expected to be more correct since the estimation error covariance was minimized and the effect of associated measurement is included in the estimations. The functionality of the Kalman filter can be boosted if a mathematical process model (like a fouling model) or even the propensity of the objective parameter is available. The latter can be obtained from the field experience or experimental data, for instance if the fouling resistance proceeds linearly, exponentially or asymptotically with time. The Kalman filter can also advance without such information but it then exclusively relies on the statistical analysis of noisy and sparse experimental results.

After each time and measurement update step, the process is repeated with the use of *a posteriori* estimations to predict the new *a priori* estimations. In the mentioned procedure, the predictions are just made for one step ahead as the width of the prediction step is set to one and the predictions at $k+1$ are depending to the estimations at time step k. If the width of the prediction step was set to $k+N$, then multiple step ahead prediction of the process states could be achieved. In this study, the performance of Kalman filter with one-step ahead predictions were investigated

THE MATHEMATICAL PROCESS MODEL

The Kalman filter depends on the accuracy of the mathematical process and the measurement models. In the absence of any mathematical process model, instead the field experience can be used which represents the propensity of fouling. In the simplest scenario, this could only be the trend for the objective function (i.e. fouling resistance), whether it proceeds linearly, exponentially or else. The mathematical process model, in the simplest practise, is the Taylor series expansion of the non-linear model of fouling process at each time step. The mathematical process model is time discrete thus the trend of fouling resistance for the first state (i.e. fouling resistance) can be expressed by:

$$R_{f_{k+1}} \;=\; R_{f_k} + \frac{dR_{f_k}}{dt}\Delta t_{k+1} \qquad (11)$$

where $\Delta t_{k+1} = t_{k+1} - t_k$.

Eq. (11) shows that the fouling resistance at time k is the summation of the one at time k and its slope at time k multiplied by the width of the time step or simply a difference term which can be negative or positive. The first order gradient of fouling resistance with respect to time, dR_{f_k}/dt, is the derivation of the above equation with respect to time, and can be written by:

$$\dot{R}_{f_{k+1}} \;=\; \dot{R}_{f_k} + \frac{d\dot{R}_{f_k}}{dt}\Delta t_{k+1} \qquad (12)$$

Eq. (12) shows that the $\dot{R}_{f_{k+1}}$ could also be written in the form of its Taylor expansion at t_k. As $\dot{R}_{f_k}$ changes with time, a new term defined here which is called the second order gradient of fouling resistance with respect to time, $d^2R_{f_k}/dt^2$ and can be expressed as:

$$\ddot{R}_{f_{k+1}} \;=\; \ddot{R}_{f_k} + \frac{d\ddot{R}_{f_k}}{dt}\Delta t_{k+1} \qquad (13)$$

Finally, the mathematical process model of the fouling process can be written in the state form as:

$$\mathbf{R}_{f_{k+1}} \;=\; \begin{pmatrix} R_f \\ \dot{R}_f \\ \ddot{R}_f \end{pmatrix}_k \;=\; \begin{pmatrix} 1 & \Delta t & 0 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{pmatrix}_k \begin{pmatrix} R_f \\ \dot{R}_f \\ \ddot{R}_f \end{pmatrix}_k \qquad (14)$$

It assumes that the trend of fouling resistance proceeds asymptotically in the form of $R_{f_{k+1}} = R_{f,\infty}(1 - \exp(-t_{k+1}/\tau))$.

It should be pointed out that the first and second order gradients of fouling resistance are not independent of the initial fouling resistance ($R_{f,t=0}$). It is though possible that the initial guess for these two values deviates much from the real values. That would, in turn, leads to the deviation of estimations from measurements for the next time step. Having said that though the adaptive method takes these deviations into consideration and updates the Kalman gain for the next time step (a new Kalman gain based on the sequence of innovations). Thus the next estimations are always being updated with statistical information on deviation of estimations from measurements leading to reliable estimation of fouling resistance.

THE MEASUREMENT MODEL

The measurement vector includes the measured fouling resistances at t_k and since there is no measurement of the first and second order gradients of fouling resistance, the two other corresponding elements will be excluded from the measurement model and thus the measurement model in the state form is:

$$\mathbf{R}_{f,meas_k} \;=\; \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} R_f \\ \dot{R}_f \\ \ddot{R}_f \end{pmatrix}_k \qquad (15)$$

INITIAL CONDITIONS

The heat exchanger is assumed to be initially clean and thus the fouling resistance at $t = 0$ is set to zero. Kalman filter, for the developed model, also considers similar propensity i.e. the fouling resistance at time zero is equal to zero. Accordingly, the estimation error covariance at $t = 0$ is zero. The process and measurement noise covariance have to be selected properly. Nevertheless, if the initial process states are unknown, these states can be substituted by random values. A better guess of process and measurement noise covariance, $\mathbf{Q}_k$ and $\mathbf{R}_k$, results in better estimations in the first time steps, however, as the new observations arrive, the filter updates these coefficients and produces related estimations.

THE EXPERIMENTAL DATA

Six sets of experimental data with asymptotic behaviour were selected. The experiments are divided into two main categories, where the first set includes the experiments that the formation of deposit layer starts immediately while in the second set, it builds up after an induction period. The sampling interval for each experiment is also considered in the model which varied between 2-3 minutes. The inlet velocity changes from 0.98 to $3.3\,\mathrm{m/s}$ and the inlet temperature varies from 94 to $360°\mathrm{C}$ where the surface temperature changes from 248 to $340°\mathrm{C}$.

The experimental data are normalized by dividing them to the maximum measured fouling resistance in each experiment. Normalization enables the qualitative comparison of various experimental results to each other as the objective function varied from zero to one regardless of initial order of magnitude of the fouling resistance. The elapsed time for each estimation step is small$(< 0.01\ \mathrm{ms})$.

IMPACT OF KALMAN GAIN AND PROCESS AND MEASUREMENT NOISE COVARIANCE

The consistency of the model is closely related to the selection of a proper Kalman gain, which depends on the process and measurement noise covariance. The Kalman gain has a direct impact on the *a posteriori* estimation of fouling resistance (see Eq. (9)). When the measurement noise covariance $\mathbf{R}_k$ reaches zero, thus the Kalman gain weights the *measurement residual* as:

$$\lim_{\mathbf{R}_k\to 0}\mathbf{K}_k = \lim_{\mathbf{R}_k\to 0}\frac{\mathbf{P}_k^-\mathbf{H}_k^T}{\mathbf{H}_k\mathbf{P}_k^-\mathbf{H}_k^T+\mathbf{R}_k} = \frac{1}{\mathbf{H}_k} \qquad (16)$$

Considering Eq. (10) again reveals when the *a priori* estimation error covariance approaches zero ($\mathbf{P}_k^-$) (process noise covaraince ($\mathbf{Q}_k$) goes to zero), thus the Kalman gain will be zero, which leads to weight the *measurement residual* ($\mathbf{H}_k\,\hat{\mathbf{x}}_k^-$) less extensively as:

$$\lim_{\mathbf{P}_k^-\to 0}\mathbf{K}_k = \lim_{\mathbf{P}_k^-\to 0}\frac{\mathbf{P}_k^-\mathbf{H}_k^T}{\mathbf{H}_k\mathbf{P}_k^-\mathbf{H}_k^T+\mathbf{R}_k} = 0 \qquad (17)$$

Whenever the measurement noise covariance reaches zero, then, the actual measurement ($\mathbf{y}_k$) is trusted more and the predicted measurement ($\mathbf{H}_k\,\hat{\mathbf{x}}_k^-$) is trusted less. As the *a priori* estimation error covariance ($\mathbf{P}_k^-$) reaches zero, the measurements will be neglected and the predicted measurement is trusted more.

INFLUENCE OF INITIAL GUESS OF THE FILTER COEFFICIENTS

The developed model is based on several assumptions; hence, it deviates to some extent from the real measurements. Thus, the selection of incorrect process and measurement coefficients will lead to deterioration of the performance of the Kalman filter for the initial time steps. It was explained in previous section that both process and measurement noise covariance ($\mathbf{Q}_k$ and $\mathbf{R}_k$) have a simultaneous effect on the *a posteriori* estimations. However, due to the adaptive character of the filter, these coefficients will be corrected in each step and finally produce optimal estimations based on new measurements.

In this study, the initial value of measurement noise covariance was set to values near zero and therefore more weights were given to measurements for the initial time steps. The initial value of the process noise covariance was set to be larger in comparison to the measurement noise covariance and therefore the predicted measurements were neglected in the primary time steps. If process prior knowledge and measurement noise covariance are available then they would help to have a better estimation of the process states in the first time steps.

ESTIMATION OF PROCESS AND MEASUREMENT NOISE COVARIANCE

The appropriate estimation of process and measurement noise covariance at each time step is a challenge. To do so, the first choice is to consider constant values for these parameters based on prior knowledge or by trial and error. Neither of these approaches though would be considered as efficient as it is time consuming to find the optimal values or it may need an excessive prior knowledge to estimate them in each time step. Furthermore the process and measurement noise covariance are not constant for all experiments and have to be defined for each run separately.

The other option to estimate the process and measurement noise covariance is to compare the elements of measurement vector with their estimated values and calculate the process and measurement noise covariance based on this knowledge [7]. The estimated residual, $\hat{\boldsymbol{\upsilon}}_k^+$, is the difference between the measurements and their *a posteriori* estimated values, and can be expressed by:

$$\hat{\boldsymbol{\upsilon}}_k^+ = \mathbf{y}_k - \mathbf{H}_k\hat{\mathbf{x}}_k^+ \qquad (18)$$

Then the measurement noise covariance can be obtained by adding an average of last M estimated residuals to the projected measurement noise covariance as:

$$\mathbf{R}_k = \frac{1}{M}\sum_{i=1}^{M}\hat{\boldsymbol{\upsilon}}_{k-i}^+\hat{\boldsymbol{\upsilon}}_{k-i}^{+\,T} + \mathbf{H}_k\mathbf{P}_k^+\mathbf{H}_k^T \qquad (19)$$

Analogous to adaptive estimation of measurement noise covariance using estimated innovation sequences, $\mathbf{R}_k$, the process noise covariance matrix, $\mathbf{Q}_k$ can be adapted based on the estimated innovation sequences as [8]:

$$\mathbf{Q}_k = \mathbf{K}_k\left(\frac{1}{M}\sum_{i=1}^{M}\hat{\boldsymbol{\upsilon}}_{k-i}^+\hat{\boldsymbol{\upsilon}}_{k-i}^{+\,T}\right)\mathbf{K}_k^T \qquad (20)$$

The larger number of samples makes the filter to become more robust as it incorporates more data into the model. The model is finally capable of handling any type of the fouling resistance trend which could be linear, asymptotic with or without induction time. The number of samples can be kept constant or increases by new measurements in each time step. The problem of such method is how to handle the first M-samples. In this paper, to overcome this problem, M was initially set to 1 and then increased until it reached 50. Then, the number of samples was kept constant. Thus any change in the trend of fouling resistance can be observed in the estimations and sudden fluctuation/oscillation of each data point would be compensated by the averaged process and measurement noise covariance.

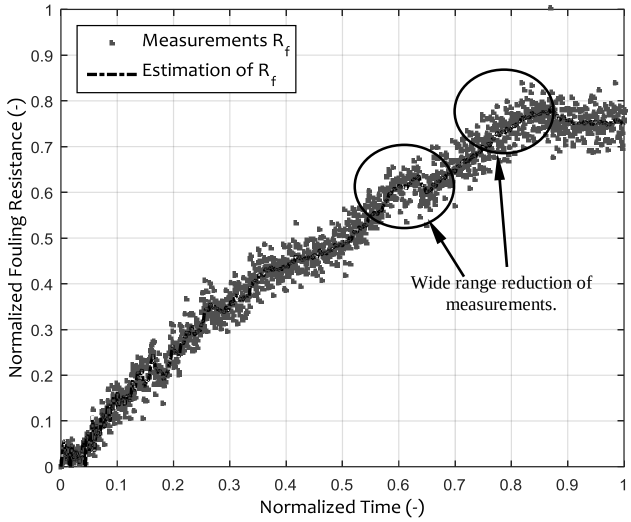

Figure 2- Simulation results for experiment No.2

RESULTS AND DISCUSSION

The simulation results are presented in Figure 1 to Figure 6. The selected experimental data are too noisy and the deposit layer in experiments No.1 to No.3 is built up immediately as soon as the experiment was started. In experiments No.4 to No. 6, though the trends of fouling resistance included an induction time, after which it follows an asymptotic behaviour. In Figure 5 and Figure 6, the trends of fouling resistance experienced negative values of fouling resistance. The developed model of fouling resistance does not contain any term for induction time and it is not capable to estimate any negative values. This introduces an additional source of uncertainty which make any mathematical process model to fail, but the developed model by calculating the process and measurement noise covariance adaptively based on estimated residuals.

In Figure 1 and Figure 3, the experimental data are too noisy. As a result, the estimations of fouling resistance initially is bound to some oscillations which are disappeared later and the estimations continue to reach an asymptotic value. In Figure 2, the trend of fouling resistance shows an asymptotic behaviour with some sudden reductions at $t = 0.65$ and $t = 0.85$. The developed model considers the wide ranging changes of the experimental measurements into the estimations as the estimations are based on M previous samples. The filter finds an optimal estimation of fouling resistance out of the measurements which are noisy and include fluctuations in each time step. In Figure 4, the experimental measurements show a dented shape for the first half of the experiments where it then continues linearly. The results show that the Kalman filter is capable of handling different trends of fouling resistance. This is due to adaptively selection of process and measurement noise covariance.

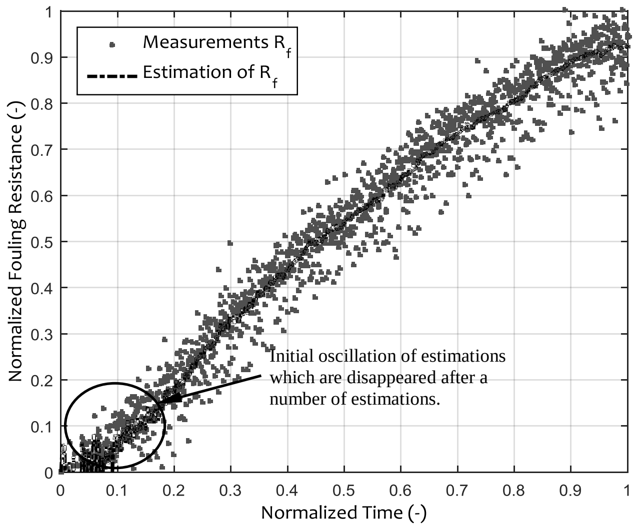

Figure 1- Simulation results for experiment No.1

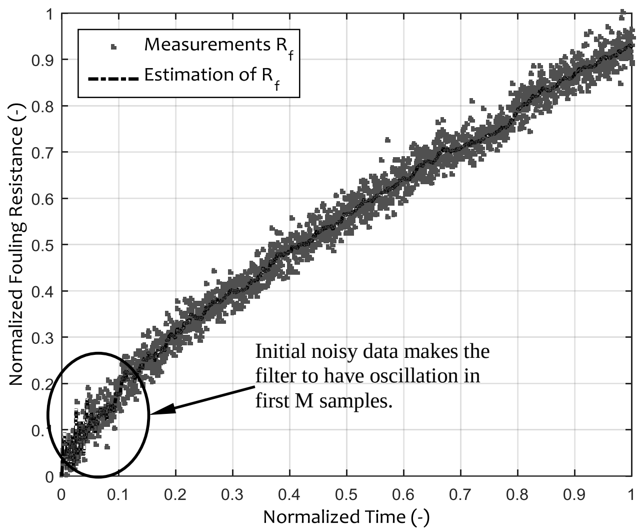

Figure 3- Simulation results for experiment No.3

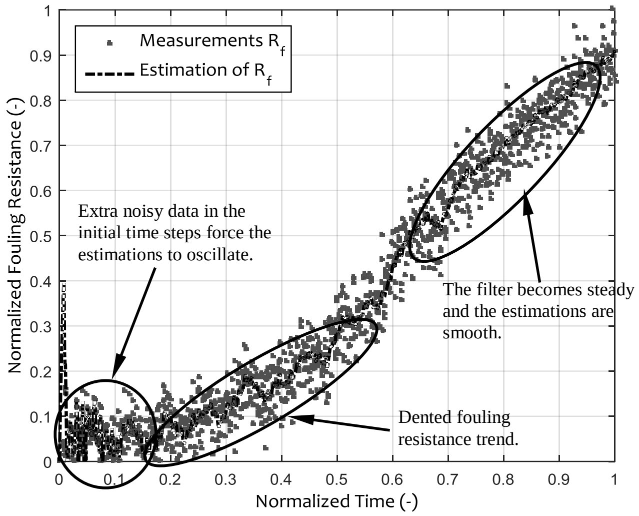

Figure 4- Simulation results for experiment No.4

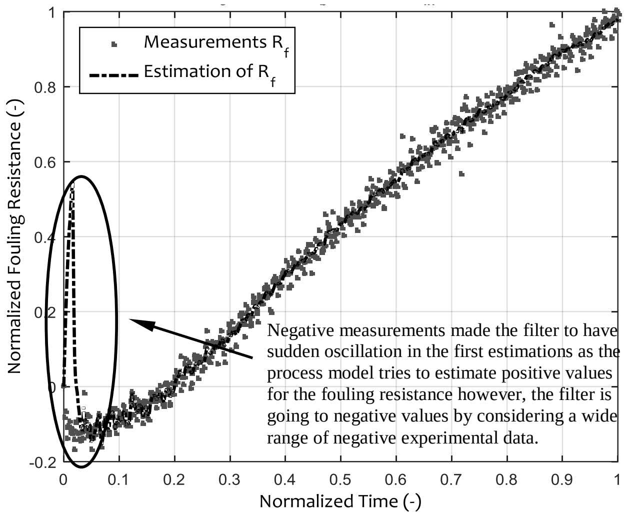

Figure 6- Simulation results for experiment No.6

In Figure 5, the trend of fouling resistance experienced an induction period where the measurements oscillates around zero value and then continues to reach an asymptotic value. The filter estimates the fouling trend both qualitatively and quantitatively. In the last simulation, Figure 6, the measurements start from negative values. A negative fouling resistance shows that the heat transfer is increased which may be due to higher heat transfer rate as a result of induced turbulence by initial deposition of foulant onto the heat transfer surface. However, as the deposit layer builds up on the heat transfer surface, it decreases the heat transfer and finally it reaches positive values. In the initial steps of this experiment, a sharp jump can be seen. This is caused as the mathematical process model attempts to make estimation with positive values, although the measurements are negative. After some measurements the filter coefficients are adapted in a way to weight the measurements more heavily than to trust the process model. Although the fact that the filter is designed to trust the measurements in the initial steps, the estimation with negative fouling resistance was out of filter's capability. In contrast to the initial mathematical process model, the filter follows the changes of negative fouling resistance.

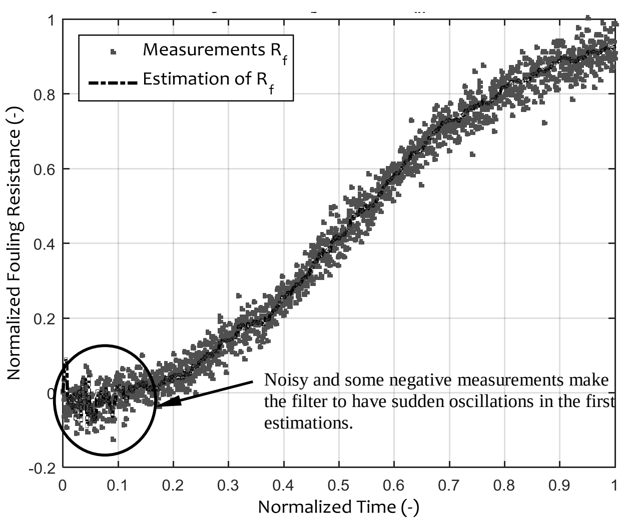

Figure 5- Simulation results for experiment No.5

For all the experiments although the experimental data are too noisy, the estimated trends are almost smooth. The estimations deviate considerably from measurements in the first steps for all simulations which disappear after a short time. This is due to stable calculation of filter coefficients due to adaptive calculation of the process and measurement noise covariance.

The best estimation of fouling resistance facilitates the determination of heat exchanger performance. This, in turn, technically means that the identified status of the heat exchanger in the upcoming steps helps to control the operating conditions in a way to avoid the critical conditions which would cause the exchanger to foul. These include malfunctioning and deteriorating the heat exchanger performance. The filter may also help to decide implementing an appropriate cleaning procedure without the need to interrupt the heat exchanger.

CONCLUSIONS

The utilization of the Kalman filter helps to analyse and model the experiments with noisy measurements. It can potentially serve as a tool to scrutinize the influence of each parameter on the deposit formation. The Kalman filter helped to minimize the error of estimation by considering the process and measurement noise covariance and the selection of process and measurement noise covariance can be adapted based on the estimated residuals based on the measurements.

NOMENCLATURE

Latin symbols

A	a $n \times n$ state transition matrix
B	a $n \times l$ input transition matrix
$\mathbb{E}$	denotes the expected value of a parameter
e	error of estimation
H	a $m \times n$ matrix of ideal connection between the measurements and the state variables at t_k
I	a $n \times n$ unit matrix
K	Kalman gain
P	estimation error covariance
Q	process noise covariance

R measurement noise covariance
$\mathbf{R}_\mathrm{f}$ the state vector for *a priori* estimation of $\mathrm{R_f}$, $\dot{\mathrm{R}}_\mathrm{f}$ and $\ddot{\mathrm{R}}_\mathrm{f}$ at each time step
$\mathrm{R_f}$ Fouling resistance, $\left[\frac{m^2K}{W}\right]$
$\dot{\mathrm{R}}_\mathrm{f}$ First order gradient of fouling resistance w.r.t. time, $\left[\frac{m^2K}{W\,s}\right]$
$\ddot{\mathrm{R}}_\mathrm{f}$ Second order gradient of fouling resistance w.r.t. time, $\left[\frac{m^2K}{W\,s^2}\right]$
t time, $[-]$
u a $\mathrm{l} \times 1$ optional input vector to control the heat exchanger and could be constant or varying at $\mathrm{t_k}$, it can be omitted
x a $\mathrm{n} \times 1$ process state vector at $\mathrm{t_k}$ (n is the number of process states at $\mathrm{t_k}$ such x, dx/dt, $\mathrm{d^2x/dt^2}$ derivations)
y measurement vector
∞ to show the asymptotic fouling resistance
$\Re$ domain of real numbers

Greek symbols
Δ difference
μ mean value of a parameter
τ fouling time constant
υ estimated residual
ω a $\mathrm{n} \times 1$ noise of mathematical process model with a covariance of $\mathbf{Q}_\mathrm{k}$ at $\mathrm{t_k}$ with normal probability distribution $p(\boldsymbol{\omega}_\mathrm{k}) \sim \mathcal{N}(0, \mathbf{Q}_\mathrm{k})$
ϑ a $\mathrm{m} \times 1$ vector of measurement noise with measurement noise covariance of $\mathbf{R}_\mathrm{k}$ at $\mathrm{t_k}$ with normal probability distribution $p(\boldsymbol{\vartheta}_\mathrm{k}) \sim \mathcal{N}(0, \mathbf{R}_\mathrm{k})$

Superscripts
ˆ the state estimation
– the minus sign for indicating the *a priori* estimation
+ the positive sign for indicating the *a posteriori* estimation

Subscripts
est estimated data point
f fouling
k point of time
meas measured data point

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