Published online www.heatexchanger-fouling.com

PREDICTION OF THE SLAGGING STATE ON COAL-FIRED BOILERS BASED ON VAGUE SETS

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ABSTRACT

The distance measures of similarity between vague sets (values) have been developed to solve the problem of slag on coal-fired boilers, and a slag-prediction model of coal burning boiler based on vague sets is proposed. In the model, six single indices and their bounds are involved. Four indices of the six indices are static indices (That is, ST, SiO₂/Al₂O₃, B/A and G) and two of the six indices are dynamic ones (that is, the dimensionless average furnace temperature ϕ_t and the dimensionless inscribed circle diameter in furnace φ_d). At the same time, the slagging performance from 10 coal burning boilers is selected as samples. The model is employed to predict the slagging performance of the four coal burning boilers from Jilin heat and power plant, Xinli power plant, Jinzhou power plant and Qinhuangdao power plant. Through predicting and determining, it proves that the accuracy of the pattern recognition model is 100 percent. Besides, compared with the normal method of pattern-recognition, it is easier for operators to predict, determine and reduce disturbance as far as possible.

INTRODUCTION

Since predicting and determining accurately the state of slagging of coal burning boilers is the bases of the combustion adjusting, energy conservation and safe operation, it has been becoming one of the hot spots of research. The techniques of predicting and determining the state of slagging of coal burning boilers have been developed in the past decades (Huang, 2004; Chen et al, 2006), and good economic returns have been obtained.

There are several factors which could cause slag on coal-fired boilers, such as coal characteristic, combustion manners, furnace gas atmosphere, pulverized coal segregation and so on. In order to predict and determine accurately, many prediction methods have been presented (Chen et al, 2006).

The theory of rough sets has been applied in the computer technology, artificial intelligence (AI), data mining, knowledge discovery (KD) etc. Vague sets is an extension form of rough sets, which has more general significance, and has expanded the description range of

object's characteristic and providing a new means of knowledge. So, in a sense, vague sets (Zadeh, 1965; Gau et al, 1999) has much more practical application value, and has attracted serious concern of academic circles and engineering circles (Chen, 1995; Szmidt and Kacprzyk, 2000). The present study tries to make use of vague sets to predict slagging performance of coal fired boiler.

VAGUE SETS

The membership function of fuzzy set assigns a number which is between zero and one for each object as the degree of membership, it not only includes the proof that the element belongs to the set, also includes the proof that the element does not belong to the set. For overcoming the insufficiency of the information by the single value description, Zadeh (Zadeh, 1975) led to go into the interval value fuzzy set in 1975, used [0,1] of inside closed subinterval to represent the degree of an element which belongs to a set its left end point means the necessity of the object which belongs to the set, the right end point means the possibility of the object belonging to the set. In 1986, Atanassov (Atanassov, 1986) considered the fuzzy set from a different angle of generalization, he adopted two number to depict a element belonging to the fuzzy set, introduced the concept of membership degree and nonmembership degree. Atanassov called the set satisfying the definition for intuitionistic fuzzy set. In 1989 the Atanassov and Gargov (Atanassov K and Gargov G, 1989) pointed out the interval value fuzzy set and intuitionistic fuzzy set are the generalization of two equivalents in the fuzzy set expansion. In 1993 the Gau and Buehrer(Gau W L, Buehrer D J, 1993) passed "the vote model" to explain the vague sets. Speak from the essence, the interval value fuzzy set, the intuitionistic fuzzy set and the vague set have no hypostatic differentiation (Deschrijver G, Kerre E E, 2003).

At the computer science and its application field, especially in the artificial intelligence, data mining and knowledge discovery in database, the theories of rough set have important of physically application, rough sets collectively describe the connection and the whole characteristic of the things, provide important tools for studying the inside contact of the things. The vague set provides a kind of new tool for knowledge representation. It scope people can know about the thing and supply a good description to the thing's attribute from the form to the contents. Both the rough set theories and vague set theories study the uncertainty problem in information system, rough sets focus on knowledge undistinguished in the information system, but the vague sets fix attention on the vagueness of the concept and imprecision of the concept person can know. However, in many situations, the concept isn't only misty, and also can't be distinguished, cause people's understanding to the concept also impossibly and completely accurate. According to this it is necessary to blend the theories of rough sets and the vague sets to make up the shortage while handling the problem alone.

Definition A vague set v in U is characterized by a truth-membership function t_v and a false-membership function f_v . Here $t_v(u)$ is a lower bound on the degree of membership of u derived from the evidence for u, and $f_v(u)$ is a lower bound on the negation of u derived from the evidence against u. Here $t_v(u)$ and $f_v(u)$ are both associated with a real number in the interval [0,1] with each element in U, where $t_v(u) \le 1$. Then

$$t_{v}: U \rightarrow [0,1] \text{ and } f_{v}: U \rightarrow [0,1]$$

$$\tag{1}$$

Suppose that $U = \{u_1, u_2, ..., u_n\}$. A vague set V of the universe of discourse U can be represented by

$$V = \{ [t_v(u_1), 1-f_v(u_1)]/u_1, [t_v(u_2), 1-f_v(u_2)]/u_2, ..., [t_v(u_n), 1-f_v(u_n)]/u_n, \}$$
(2)

where $t_v(u_i) \leq \mathbf{m}_i(u_i) \leq 1 - f_v(u_i), \forall u_i \in U \text{ and } 1 \leq i \leq n$.

This approach bounds the degree of membership of u to a subinterval $[t_v(u), 1-f_v(u)]$ of [0,1]. In other words, the exact degree of membership $\mathbf{m}_{u}(\mathbf{u})$ of u may be unknown, but is bounded by $t_v(u) \le \mathbf{M}_v(u) \le 1 - f_v(u)$, where $t_v(u) + f_v(u)$ ≤ 1 . The precision of the knowledge about u is characterized by the difference $1-t_v(u)-f_v(u)$. If the difference is small, the knowledge about u is relatively precise, if it is large, we know correspondingly little. If $t_v(u)$ is equal to $1-f_v(u)$, the knowledge about u is exact, and the vague set reverts back to a fuzzy set. If $t_v(u)$ and $1-f_v(u)$ are both equal to 1 or 0, depending on whether u does or does not belong to V, the knowledge about u is very exact and the vague set reverts back to an ordinary set. For example, the fuzzy $set{0.6/u}$ can be represented as the vague set {[0.6, 0.6]/u}, while the ordinary set{u} can be represented as the vague set $\{[1,1]/u\}.$

The following is the degree of nearness between two vague subsets. Assuming there are n vague subsets A_1 , A_2, \ldots, A_n on U, if there is $s \in \{1, 2, \ldots, n\}$ satisfying $\sigma(B, A_s) = \max\{\sigma(B, A_1), \ldots, \sigma(B, A_n)\}$. Then B is closest to A_s , where $\sigma(B, A_s)$ is called nearness of vague sets B and A_s . If A_1, \ldots, A_n stand for n known patterns, B stands for a unknown pattern. When B is closest to A_s , then pattern B should be fell under the pattern class which includes pattern

is clearly to give the representation of the degree and A_s . Assumed that every pattern has n characteristic indexes, called $u_1, u_2, ..., u_n$ and represented as $u = \{u_1, u_2, ..., u_n\}$, each characteristic index describes certain aspect of the pattern, thus n known patterns can be represented as follows:

$$A = \sum_{i=1}^{n} [t_A(u_i), 1 - f_A(u_i)]/u_i$$
(3)

Unknown pattern B can be represented as follows:

$$B = \sum_{i=1}^{n} [t_{B}(u_{i}), 1 - f_{B}(u_{i})]/u_{i}$$
(4)

Assume $V_A(u_i) = [t_A(u_i), 1 - f_A(u_i)]$ stand for the membership value of u_i in A; $V_B(u_i) = [t_B(u_i), 1 - f_B(u_i)]$ stand for the membership value of u_i in B, so the scores of A and B:

$$S(V_A(u_i)) = t_A(u_i) - f_A(u_i)$$
 (5)

$$S(V_B(u_i)) = t_B(u_i) - f_B(u_i)$$
 (6)

where i=1,2,...,n. Assume that the weight of feature u_i in a pattern is ω_i , $\omega_i \in (0,1]$, $1 \le i \le n$, the weighted similarity measure between the two vague patterns A and B can be evaluated by the function T':

$$\begin{split} T'(A,B) &= \sum_{i=1}^{n} \omega_{i} M'(V_{A}(u_{i}), V_{B}(u_{i})) / \sum_{i=1}^{n} \omega_{i} \\ &= \sum_{i=1}^{n} \omega_{i} |1 - [S(V_{A}(u_{i}) - S(V_{B}(u_{i}))] / 4 - [|t_{A}(u_{i}) - t_{B}(u_{i})| + |f_{A}(u_{i}) - f_{B}(u_{i})|] / 4 | / \sum_{i=1}^{n} \omega_{i} \quad (7) \end{split}$$

We first set threshold value α , $\alpha \in [0,1]$, when $T \ge \alpha$, it indicates that the pattern to be recognized satisfies the precision of recognition.

Measures of similarity between vague values and sets have been studied in many literatures. Chen (1995) and Szmidt (2000) had given the methods of measures of similarity between vague values and sets. The following to come will propose a similarity measure between vague sets.

SIMILARITY MEASURE BASED ON THE SENSE OF DISTANCE

In the general sense, similarity measure is closely related to distance, thus, it is provided with basic meaning by using the form of "Similarity measure = 1 - Distance measure (D)" to evaluating the value of similarity. For any vague values x_i , we have

$$m_{v}(x_{i}) = 1 - t_{v}(x_{i}) - f_{v}(x_{i})$$
(8)

Thus, it has constituted a space plane equation. Let (0, t, f, m) be the Cartesian coordinates in space of the plane, and "o" be the origin, so the vague sets is in the plane determined by points (1,0,0), (0,1,0) and (0,0,1). Let $x = [t_x, 1-f_x]$ and $y = [t_y, 1-f_y]$ be two vague sets in U, and A(t_x , f_x , m_x) and B(t_y , f_y , m_y) are the two corresponding points in the Cartesian coordinates in space. The Euclidean distance between the two points can be evaluated by d(A, B),

$$d(A,B) = \sqrt{(t_x - t_y)^2 + (f_x - f_y)^2 + (m_x - m_y)^2}$$
(9)

Because t, f and m are non-negative, the max Euclidean distance in the Cartesian coordinates in space is the side of the triangle constituted by the three points (1,0,0), (0,1,0) and (0,0,1). That is maxd(A, B) = $\sqrt{2}$. Let $x = [t_x, 1-f_x]$, $y=[t_y, 1-f_y]$ be vague sets in U, where $t_x \in [0,1]$, $f_x \in [0,1]$, $t_y \in [0,1]$, $t_y \in [0,1]$, $t_x + f_y \le 1$, $t_y + f_y \le 1$, $m_x = 1 - t_x - f_x$, $m_y = 1 - t_y - f_y$. Thus the similarity measure between the vague sets x and y can be evaluated by the function M_R :

$$M_{R}(x, y) = 1 - d$$
 (10)

Where

$$d = \sqrt{\frac{(t_x - t_y)^2 + (f_x - f_y)^2 + (m_x - m_y)^2}{2}}$$
(11)

PREDICTION OF SLAG ON COAL-FIRED BOILERS BASED ON SIMILARITY MEASURE ON THE SENSE OF DISTANCE

Here, six single indices are selected, which are high in prediction accuracy, and used as criterions. They are Softening Temperature (ST), SiO₂-Al₂O₃ ratio (SiO₂/Al₂O₃), Alkali-acid ratio (B/A), Silicon percentage of value (G), the dimensionless average furnace temperature (ϕ_t) and the dimensionless inscribed circle diameter in furnace (ϕ_d). At the same time, the slagging performance of ten typical coal-fired boilers from the references (Wen, 2006; Wen et al, 2007) is taken as the samples. The slagging characteristics of four coal-fired boilers in Jilin heat and power plant, Xinli power plant, Jinzhou power plant and Qinhuangdao power plant are predicted by using this method. The four boilers are named as B1, B2, B3 and B4. The prediction steps are

Table 1	Data	. f . J	ام میں میں مرکب ا		- 1	ار ما مسمعه ار	hailana
Table 1.	Data (ог а	letermined	and	to be	determined	boners

indices samples	ST	B/A	SiO ₂ /Al ₂ O	G (100%)	ϕ_t	ϕ_{d}	Slagging state
A1	1500	0.08	1.32	88.67	0.910	0.62	minor
A2	1460	0.12	1.64	86.80	0.950	0.49	minor
A3	1420	0.18	1.6	80.24	0.960	0.41	minor
A4	1500	0.12	1.76	85.68	0.980	0.52	minor
A5	1190	0.36	2.56	67.44	1.120	0.50	serious
A6	1120	0.22	3.29	78.00	1.110	0.50	serious
A7	1205	0.25	3.47	77.69	1.090	0.61	serious
A8	1290	0.22	2.68	78.90	1.130	0.54	middle
A9	1160	0.16	4.25	59.20	1.120	0.48	middle
A10	1026	1.28	3.16	79.87	1.030	0.37	middle
B1	1190	0.51	0.32	63.60	1.190	0.64	to be determined
B2	1450	0.17	1.83	80.80	0.960	0.51	to be determined
B3	1440	0.17	2.76	84.83	1.080	0.62	to be determined
B4	1400	0.22	1.50	82.44	1.040	0.49	to be determined

as followed:

Step 1: The establishment of collection of knownslagging coal-fired boiler samples. The sample collection m of known-slagging coal-fired boilers, where $m=(x_1, x_2, \dots, x_n)$, $n \in N$, has been established. The knownslagging coal-fired boilers samples are shown in table 1.

Step 2: Calculation of t_x , f_x and m_x

To calculate t_x , f_x and m_x , the fuzzy membership function is introduced, which is shown in fig.1. The

thresholds of the six indices are shown in table 2(Yang , 2002).

Generally, let t_x be minor level of slagging, f_x be middle and m_x be serious. The assumption is arbitrary and it would not impact on the results of evaluation. Take the single indice "ST" from A8 as an example, because 1290 °C is



Fig1. Membership grade function

between 1330° C(V_{center}) and 1260° C(V_{down}), so t_x(minor level) is 0.0, f_x (middle level) is 0.429 and m_x (serious level) is 0.571 according to the equations as follows.



Table2. Thresholds of the six indices

thresholds indices	\mathbf{V}_{up}	V_{down}	V _{center}
ST(℃)	1390	1260	1330
B/A	0.206	0.400	0.350
SiO ₂ /Al ₂ O ₃	1.87	2.65	2.10
G (100%)	78.8	66.1	70.0
ϕ_t	0.970	1.065	1.050
ϕ_{d}	0.4750	0.5875	0.5300

The results are shown in table 3. Here we would focus on how to calculate the dimensionless average furnace temperature and the dimensionless inscribed circle diameter in furnace. All the other indices can be found from the references (Harbin Puhua coal combustion technology development center, 2002).

(1) The dimensionless inscribed circle diameter in furnace $(d_{ic}\!/D)$

The dimensionless inscribed circle diameter in furnace, which is defined as the ratio of the inscribed circle diameter d_{ic} (Harbin Puhua coal combustion technology development center, 2002) to the equivalent width D, has a great influence on the furnace aerodynamic performance. The equivalent width is given as

$$\mathbf{D} = \sqrt{\mathbf{a}\mathbf{b}} \tag{12}$$

where a is the depth of the furnace and b is the width of the furnace, m.

If the value of d_{ic}/D is bigger than needed, the airflow would deflect and the flame is easy to stick to the furnace wall. If the pulverized coal in the airflow is in melt state, it would be easily slagged on the water wall. The threshold of the dimensionless inscribed circle diameter in furnace is given as followed:

d _{ic} /D<0.4750	minor degree of slagging
$0.4750 \leq d_{ic}/D \leq 0.5875$	middle degree of slagging
d _{ic} /D>0.5875	serious degree of slagging

(2) The dimensionless average furnace temperature

The furnace temperature plays an important role in slagging. The temperature value has a big effect on particle state on the water wall. The boiler load distribution curve has been drawn based on the test data of the furnace temperature field from 22 high-capacity boilers made by China and on the data of high-capacity boilers gained by CE U.S. Company. The distribution curve reflects the relationship among the maximum furnace temperature t_{max} , the average furnace temperature t_{at} and cross-section heat load q_{hl} , where the relationship between t_{at} and q_{hl} (Harbin Puhua coal combustion technology development center, 2002) is:

$$t_{at} = 1144 + 2491 \ln(0.86q_{hl}), \ ^{\circ}C$$
 (13)

During the process of the slagging formation, if $t_{at} < ST$, then ash particles in airflow does not melt, the possibility that the ash particles slagged on the water wall is very small; if $t_{at} > ST$, then the ash particles is in melt state and easy to stick to the furnace wall. For this reason, the dimensionless average furnace temperature (t_{at}/ST) is introduced to predict slagging degree:

t _{at} /ST<0.970	minor degree
$0.970 \le t_{at} / ST \le 1.065$	middle degree
t _{at} /ST>1.065	serious degree

Step 3: Pattern recognition of the boilers to be predicted by using the .eq. 10.

The results of pattern recognition are shown in table 4. Take B2(Xinli power plant) as an example, the similarity degree between each sample and B2 is 0.687, 0.773 and so on, and the max one is 0.782. So, we could predict that the state of slagging of the boiler in Xinli power plant is minor, which agrees with its actual extent of slagging: minor.

indices samples	ST	B/A	SiO ₂ /Al ₂ O ₃	G(100%)	ϕ_t	ϕ_{d}
A1	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(0,0,1)
A2	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(0.727,0.273,0)
A3	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)
A4	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(0.875,0.125,0	(0.182,0.818,0)
A5	(0,0,1)	(0,0.8,0.2)	(0,0.164,0.836	(0,0.656,0.344	(0,0,1)	(0.545,0.455,0)
A6	(0,0,1)	(0.903,0.097,0	(0,0,1)	(0.909,0.091,0	(0,0,1)	(0.545,0.455,0)
A7	(0,0,1)	(0.694,0.306,0	(0,0,1)	(0.874,0.126,0	(0,0,1)	(0,0,1)
A8	(0,0.429,0.571	(0.903,0.097,0	(0,0,1)	(1,0,0)	(0,0,1)	(0,0.826,0.174)
A9	(0,0,1)	(1,0,0)	(0,0,1)	(0,0,1)	(0,0,1)	(0.909,0.091,0)
A10	(0,0,1)	(0.486,0.514,0	(0,0,1)	(1,0,0)	(0.25,0.75,0)	(1,0,0)
B1	(0,0,1)	(0,0,1)	(1,0,0)	(0,0,1)	(0,0,1)	(0,0,1)
B2	(1,0,0)	(1,0,0)	(1,0,0)	(0,0, 1)	(1,0,0)	(0.364,0.636,0)
B3	(1,0,0)	(1,0,0)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,1)
B4	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(0.125,0.875,0	(0.727,0.273,0)

Table 3. t_x , f_x and m_x of the sample boilers

Table 4. Results of pattern recognition of the boilers

Samples To be predicted	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	Actual extent of Slagging (samples to be predicted)
B1	0.333	0.184	0.167	0.186	0.458	0.203	0.528	0.298	0.507	0.205	serious
B2	0.687	0.773	0.727	0.782	0.220	0.294	0.146	0.286	0.409	0.183	minor
В3	0.500	0.351	0.333	0.356	0.399	0.513	0.625	0.535	0.674	0.431	Serious biased
B4	0.705	0.854	0.812	0.784	0.191	0.448	0.339	0.406	0.312	0.515	minor
Slagging state of sample boiler	minor	mino r	mino r	mino r	serious	serious	serious	middl e	middl e	middl e	

COMPARISON

To evaluate the validity of the method, which has been developed in the paper (the second method), is compared with the method introduced by Wen (2006)(The following called method one). The comparison of the results is shown in fig. 2. There are ten groups of columns in fig. 2. The left light-colored one in each group of columns represents the result by using the method one and the right dark one represents the results based on the second method. The length of columns represents the similarity degree. The longer the length is, the higher the similarity degree is. It can be concluded from the fig. 2 that:

(1) the second method is reasonable and feasible. For the four boilers, results of the prediction agree with the actual extent of slagging.

(2) the second method is more accurate than method one, i.e. although prediction result of Qinhuangdao power plant agrees with the actual extent of slagging in fig. b, we could easily find out that the data: 0.512 (minor), 0.510 (middle), 0.509 (serious) from method one, are more or less the same with one another, so operators are difficult to predict the slagging state based on the data. But the second method is not the case; the operators could easily predict the slagging state of the coal-fired boilers based on this method.

PREDICTION SYSTEM

Based on the method, an on-line prediction system has also been developed by object-oriented software—Delphi. This system could not only monitor boiler operation status and predict the slagging state, but also expand the quantity of boiler samples, allowing the operators to add typical boiler samples to the database at any time.

When the operators have access to the six single indices through chemical analysis and calculation, and input them to





c. results comparison (Jinzhou)



d. results comparison (Qinhuangdao)

Fig2. Comparison of the boilers' results

the prediction system, they could predict the slagging state of the boiler easily. Here, we only have 10 typical coal-fired boilers, so, in order to improve the accuracy of the prediction, it is necessary for operators to expand the database capacity of typical coal-fired boilers. So, the online prediction system provides operators with such functions and operators could increase known-slagging coal-fired boiler samples easily.

Besides, this system has the function of data processing. Take the Xinli power plant for example, if adding respectively the data in accordance with minor, middle and serious degree of slagging, and averaging the sums

	A1-4	A5-7	A8-10	Actual extent of slagging (To be predic -ted)
B1(Jidian)	0.218	0.396	0.337	serious
B2(Xinli)	0.742	0.220	0.293	minor
B3(Jinzhou)	0.385	0.546	0.512	serious biased
B4(Qin Huang-dao)	0.789	0.411	0.326	minor
Actual extent of slagging (samples)	min	ser	mid	

Table5. Results by means of average

respectively, the results are 0.742(minor degree of slagging), 0.220(middle degree of slagging) and 0.293(serious degree of slagging), so the results of Xinli power plant is minor degree of slagging. All the other results are shown in table 5. It is obvious that the conclusions here agree with actual extent of slagging. So, the data processing is reasonable and feasible.

CONCLUSIONS

Vague sets is an extension form of rough sets, which has more general significance, and has expanded the description range of object's characteristic and providing a new means of knowledge. Here, distance measures of similarity between vague sets(values) have been developed to solve the problem of slag on coal-fired boilers. In the model, six single indices: 4 static norms(ST, SiO₂/Al₂O₃, B/A and G) and 2 dynamic ones(f_t and f_d), are selected, which are high in prediction accuracy, and used as criterions. Through prediction of the four coal burning boilers, it proves that the new pattern recognition model built in this paper is reasonable and feasible. Compared with the normal method of pattern-recognition, it was easier for operators to predict, determine and reduce disturbance as far as possible.

An on-line prediction system has also been developed by object-oriented software. The system has the function of data processing. From the data processing results, we can come to the conclusion: if the data is added respectively in accordance with minor, middle and serious degree of slagging and averaging the summations, the accuracy is still 100 percent.

ACKNOWLEDGEMENT

This project Supported by the National Basic Research Program(973 Program) (2007CB206904) and Jilin Province Science and Technology Development Project (20070529)

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