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EFFECT OF DEPOSIT FORMATION ON THE PERFORMANCE OF ANNULAR FINNED TUBES DURING NUCLEATE POOL BOILING

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ABSTRACT

Low finned tubes can potentially mitigate crystallization fouling. However the extent of their performance is still a matter of substantial investigation. In this study, a numerical model has been developed to simulate the effect of deposits on the thermal performance of finned tubes that occurred during pool boiling, in terms of fin temperature profile and thermal efficiency. An analytical solution of the one-dimensional heat conduction equation for an annular fin was sought for steady state and transient operation (fouling). The temperature distribution across the fin as well as the fin efficiency were calculated for both clean and fouling conditions. The numerical results showed that the presence of fouling increases the base temperature and hence the temperature distribution along the fin, consequently decreasing the boiling heat transfer coefficient. Furthermore, the model underlines important features of fins such as their increased efficiency when fouling occurs. As explained numerically, this is related to the improved temperature uniformity in the fins in the event of fouling

1. INTRODUCTION

Heat transfer during nucleate boiling is highly efficient and thus extensively utilized in industry for a wide range of applications. The superiority of nucleate boiling over convective heat transfer lies in the bubble behaviour that generates additional turbulence, leading to significantly higher heat transfer performance. Consequently, a higher rate of heat can be transferred for the same heat transfer area in nucleate boiling compared to convective heat transfer. Nevertheless, this advantage may be diminished if the heat transfer surface is subject to fouling.

Whilst fouling is already a chronic industrial problem for convective heat transfer, it is even more severe under boiling conditions. In case of crystallization fouling, for instance, this is due to evaporation and bubble formation which increase the salt concentration in the micro-layer beneath the bubbles (Müller-Steinhagen and Jamialahmadi, 1990). As a result, scale formation during boiling is usually characterized by rapid growth of deposit with an associated drop in heat transfer coefficient.

In the past few decades, a wide rang of structured heat transfer surfaces have been developed to enhance boiling heat transfer to pure liquids. Furthermore it has also been reported that some of these structured surfaces may potentially mitigate fouling (Somerscales and Curcio, 1990; Bergles and Somerscales, 1995; Thomas, 1997).



Fig. 1 Comparison of fouling resistances for plain stainless steel and annular finned Cu-Ni tubes at 100 and 300 kW/m² for a CaSO₄ concentration of 1.6 g/L (Esawy, 2011).

Most recently, Esawy et al. (2010) investigated the effect of various low finned tubes on crystallization fouling during boiling of $CaSO_4$ solutions. For instance, Fig. 1 compares the fouling resistance for a finned copper-nickel

tube and a plain stainless steel tube for a $CaSO_4$ concentration of 1.6 g/L and heat fluxes of 100 kW/m², and 300 kW/m². For 300 kW/m², the fouling resistance of the finned tube increases slowly with time until it reaches an asymptotic value of $1x10^{-5}$ m²K/W after about 1200 minutes. For the plain tube, the fouling resistance increases rapidly until the surface temperature of the heater reaches its maximum set-value of $1.47x10^{-4}$ m²K/W. It is evident that the fouling resistance for the same time of operation could be reduced by 93% by using a finned tube instead of a plain one.



Fig. 2 Relative drop of heat transfer coefficient versus heat flux at time of 1350 min for 19 and 26 fpi annular finned tubes and a smooth tube, for a $CaSO_4$ concentration of 1.6 g/L (Esawy et al, 2010).

Fig. 2 provides more details about the performance of finned tubes compared to a plain tube under fouling conditions, as well as about the effect of fin density, i.e number of fins per inch (fpi). Data included in both figures have been obtained after 1350 min of experimental time. At this time, the finned tubes have reached the asymptotic fouling resistances for the investigated heat fluxes. For the plain tube, however, an asymptotic fouling resistance has only been found for a heat flux of 100 kW/m²; for higher heat fluxes the fouling runs were discontinued once the surface temperature exceeded its maximum set value. The overall boiling heat transfer coefficient for the attempted finned tubes is calculated considering its root surface area. More results and details can be found in Esawy et al. (2010).

Furthermore, Fig. 2 shows a strong reduction of heat transfer coefficient by 50% and 75% for heat fluxes of 100 and 300 kW/m² for the plain tube. For the finned tubes, only 2% reduction occurred for the low heat flux of 100 kW/m². For higher heat fluxes of 200 and 300 kW/m², the 26 fpi tube experienced the smallest reduction of 15% and 8%, respectively, while the 19 fpi tube had a still moderate reduction of 25% and 20%, respectively.

For utilization of various finned tubes in boiling applications with fouling, not only the fouling resistance (R_f) and the overall heat transfer coefficient (α_f) have to be predicted, but also the thermal performance of the fins, i.e. the thermal efficiency and the temperature distribution. This is because fouling on finned tubes will undoubtedly also affect the thermal performance of the fins.

The present study endeavors to develop a mathematical model for clean and fouling one-dimensional heat conduction in an annular fin. The temperature distributions along the fin as well as the fin efficiency were calculated for both clean and fouling conditions.

2. MATHEMATICAL MODEL DEVELOPMENT

One major consideration for the prediction of heat transfer during boiling on finned surfaces is the determination of fin efficiency and temperature distribution, even for clean conditions. The nucleate boiling heat transfer coefficient varies along the fin length as it depends strongly on the local wall superheat. As a result, the prediction of fin efficiency as well as temperature distribution requires an iterative, stepwise calculation by computer. Only a few models are available in the open literatures for fin performance calculations under fouling conditions (Epstein and Sandhu, 1978; Middis and Müller-Steinhagen, 1991; Shilling, 1994; Aparajith et al, 2001). However, these models are for fouling under convective heat transfer rather than nucleate boiling.



Fig. 3 Geometry of the simulated annular fin.

Fig. 3 shows the shape of a simulated annular fin with all its geometrical parameters. The assumptions made for the model are:

- 1. The analyzed fin is thin with a constant thermal conductivity, so that one-dimensional radial heat conduction across the fin can be assumed.
- 2. The temperature at the fin base is uniform.
- 3. The temperature of the fin does not change with time unless it is prone to fouling.
- 4. An adiabatic fin tip, i.e., no heat transfer from the tip.

- 5. The time step during modelling is very small so that the variation in the surface temperature within this interval is negligible.
- 6. The boiling heat transfer coefficient along the fin height depends on the local wall superheat ΔT .

2.1 Formulation of the model

Considering the above-mentioned assumptions, the conservation of energy for the simulated annular fin yields the following differential equation as a function of fin temperature:

$$r^{2}\frac{d^{2}T}{dr^{2}} + r\frac{dT}{dr} - \frac{2\alpha}{k_{f}s_{f}}r^{2}(T - T_{b}) = 0$$
(1)

or

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} - \frac{2\alpha}{k_f s_f}(T - T_b) = 0$$
(2)

where:

- k_f fin thermal conductivity, W/m.K
- s_f mean fin thickness, m
- T local fin temperature, K
- T_b bulk temperature, K
- α average heat transfer coefficient (at the base), W/m²K

Eq. (2) may be reformulated with non-dimensional groups as follows

$$X = \frac{r - r_i}{r_o - r_i} \qquad \text{then, } dX (r_o - r_i) = dr \qquad (3)$$

where r_i is the inner diameter of the fin (i.e. the fin base), and r_o is the fin outer diameter (i.e. the fin tip).

$$\theta = \frac{T - T_b}{T_{fb} - T_b} \qquad \text{then } d\theta \left(T_{fb} - T_b \right) = dT \qquad (4)$$

where T_{fb} is the fin temperature at the base i.e. at $r = r_i$.

The fin parameter mL can be defined as:

$$(mL)^{2} = \frac{2\alpha(r_{o} - r_{i})^{2}}{k_{f}s_{f}}$$
(5)

The ratio of the inner radius of the fin to the fin height can be defined as:

$$B = \frac{r_i}{r_o - r_i} \tag{6}$$

Typical values for r_i and r_o were 6.35 and 7.85 mm respectively. Substituting in Eq. (2) yields

$$\frac{d^2\theta}{dX^2} + \frac{1}{X+B}\frac{d\theta}{dX} - (mL)^2\theta = 0$$
(7)

Equation (7) is the general equation for an annular fin under clean convective heat transfer conditions. For constant heat flux, if fouling occurs on the finned tube, then the temperature difference ΔT increases and thus the average heat transfer coefficient decreases. The fouling resistance R_f can be expressed in terms of clean and fouled heat transfer coefficients as:

$$R_f = \frac{1}{\alpha_f} - \frac{1}{\alpha_c} \tag{8}$$

where α_c and α_f are the clean and fouled heat transfer coefficients, respectively. Eq. (8) can be rearranged to predict the fouled heat transfer coefficient as follows:

$$R_f = \frac{\alpha_c - \alpha_f}{\alpha_c \alpha_f} \tag{9}$$

$$\alpha_f = \frac{\alpha_c}{1 + \alpha_c R_f} \tag{10}$$

which can be written as:

$$\alpha_f = \frac{\alpha_c}{1 + Bi_f} \tag{11}$$

where "Bif" is the fouling Biot number as

$$Bi_f = \alpha_c R_f \tag{12}$$

 Bi_f is a measure of the fouling level on a heat transfer surface. For instance, Bi_f of zero represents clean condition and 4 when fouling is severe (see Fig. 1). By substituting Eq. (12) into (7) then

$$\frac{d^2\theta}{dX^2} + \frac{1}{X+B}\frac{d\theta}{dX} - \frac{(mL)_c^2}{1+Bi_f}\theta = 0$$
(13)

where $(mL)_c$ is the clean fin parameter which is based on the clean heat transfer coefficient α_c .

The corresponding boundary conditions are shown in Fig. 4 and can be defined as:



Fig. 4 Boundary conditions for the simulated fin.

Eq. (13) with its boundary conditions is a general equation valid only for convective heat transfer mode, in which the heat transfer coefficient does not change along the fin height. For boiling heat transfer, the temperature gradient along the fin leads to a change of the heat transfer coefficient in longitudinal direction. The boiling heat transfer coefficient α_{fin} along the fin is a function of the local fin superheat according to the following equation (Lai and Hsu, 1967; Liaw and Yeh, 1994):

$$\alpha_{fin} = \alpha_b \theta^n \tag{14}$$

where α_b is the heat transfer coefficient at the fin base. The values for "n" depend on the boiling mode and vary between 0, 2, and -0.25 for convective, nucleate and film boiling, respectively (Chowdhury and Hashim, 2006). For nucleate pool boiling with n =2, Eq. (13) can be re-written as:

$$\frac{d^2\theta}{dX^2} + \frac{1}{X+B}\frac{d\theta}{dX} - \frac{(mL)_c^2}{1+Bi_f}\theta^3 = 0$$
(15)

Eq. (15) is the general equation for the annular fin under nucleate boiling conditions.

2.2 Solution procedures

Eq. (15) is a nonlinear, second order ordinary differential equation (ODE) which cannot be solved analytically. However, Eqs. (13) (for convective heat transfer) and (15) (for nucleate boiling heat transfer) can be solved numerically for the above boundary conditions using the Runge-Kutta-Method, by transferring the second order ODEs to a set of first order ODEs as follows:

If
$$\frac{d\theta}{dX} = Y$$
 and $n' = n+1$ (16)

then

$$\frac{dY}{dX} + \frac{1}{X+B}Y - \frac{(mL)_c^2}{1+Bi_f}\theta^{n'} = 0$$
(17)

at X=0 $\theta = \theta_b = 1$ at X=1 $d\theta/dX = Y = 0$ (adiabatic fin tip)

The first order ODEs (16) and (17) can be solved simultaneously for the above boundary conditions to find the temperature distribution $\theta(X)$ along the fin. A computer program was written to solve these equations numerically using a backward Runge-Kutta-Method together with the Newton-Raphson iteration method to predict the initial condition at the fin base.

3. FIN EFFICIENCY CALCULATION

An important parameter for the fin thermal performance is the fin efficiency η_{fin} which is the ratio between the actual fin heat transfer rate q_{fin} and the maximum fin heat transfer rate q_{max} which can be defined as:

$$\eta_{fin} = \frac{q_{fin}}{q_{\max}} \tag{18}$$

where

$$q_{fin} = -kA_c \frac{dT}{dr}|_{r=r_i}$$
(19)

Referring to Fig. 3, the fin cross-sectional area A_c at the fin base can be calculated as:

$$A_c = 2\pi r_i s_f \tag{20}$$

and

$$\frac{dT}{dr}|_{r=r_i} = \frac{T_{fb} - T_b}{r_o - r_i} \frac{d\theta}{dX}|_{x=0}$$
(21)

Therefore,

$$q_{fin} = -\frac{2\pi r_i s_f k_f}{r_o - r_i} (T_{fb} - T_b) \frac{d\theta}{dX}|_{x=0}$$
(22)

The maximum fin heat transfer rate can be defined as:

$$q_{\max} = \alpha A_{fin} (T_{fb} - T_b)$$
⁽²³⁾

in which the outside fin surface area A_{fin} is calculated as

$$A_{fin} = 2\pi (r_o^2 - r_i^2)$$
(24)

ignoring the area at the fin tip. Therefore,

$$q_{\max} = 2\pi \,\alpha (r_o^2 - r_i^2)(T_{fb} - T_b)$$
(25)

Then

$$\eta_{fin} = \frac{2r_i}{(r_o - r_i)(r_o^2 - r_i^2)} \left(\frac{s_f k_f}{2\alpha}\right) \frac{d\theta}{dX} |_{x=0}$$
(26)

which leads to:

$$\eta_{fin} = \frac{2 r_i}{(mL)^2 (r_o + r_i)} \frac{d\theta}{dX} \Big|_{x=0}$$
(27)

Eq. (27) can be applied to define the clean and fouled fin efficiencies $(\eta_{fin})_c$ and $(\eta_{fin})_f$ as follows:

$$(\eta_{fin})_{c} = \frac{2r_{i}}{(ml)_{c}^{2}(r_{o} + r_{i})} \left(\frac{d\theta}{dX}|_{x=0}\right)_{c}$$
(28)

$$(\eta_{fin})_{f} = (1 + Bi_{f}) \frac{2 r_{i}}{(ml)_{c}^{2} (r_{o} + r_{i})} \left(\frac{d\theta}{dX}\Big|_{x=0}\right)_{f}$$
(29)

Where the subscripts c and f refer to clean and fouled conditions, respectively. The efficiency ratio $\frac{(\eta_{fin})_f}{(\eta_{fin})_c}$ can be calculated from:

$$\frac{(\eta_{fin})_f}{(\eta_{fin})_c} = (1 + \alpha_c R_f) \frac{\left(\frac{d\theta}{dX}|_{x=0}\right)_f}{\left(\frac{d\theta}{dX}|_{x=0}\right)_c}$$
(30)

The values of θ and $\frac{d\theta}{dX}|_{x=0}$ for clean and fouling

conditions are obtained from the output of the developed computer program to calculate the efficiency ratio. A Q-Basic computer program was written to calculate these parameters.

4. MATHEMATICAL RESULTS AND DISCUSSION

Referring to Eq. (17), which is the general equation of the numerical model, the main non-dimensional input parameters are:

- 1. The index "n'" defines the mode of heat transfer i.e. n'=1 for convective boiling and n'=3 for nucleate boiling.
- 2. The fin parameter "mL" defines the effect of geometrical and thermo-physical parameters such as height, thickness, and thermal conductivity of the fin. The term "(mL)_c" is determined when the fin is still clean, i.e. using a clean heat transfer coefficient.
- 3. "Bi_f" defines the level of fouling on the fin; i.e. $Bi_f = 0$ means clean condition.

Accordingly, the main outputs of the model are:

- 1. the clean and fouled temperature distribution along the fin $\theta(x)$;
- 2. the clean and fouled thermal efficiencies of the fin η_c , η_f ; and

3. the efficiency ratio
$$\frac{(\eta_{fin})_f}{(\eta_{fin})_c}$$

4.1 Validation of Numerical Solutions

The analytical solution of the temperature distribution and fin efficiency for clean convective heat transfer was given by Incropera and Dewitt (2001) and Mills (1999) as follows:

$$\theta = \frac{\theta}{\theta_b} = \frac{I_o(mr)K_1(mr_o) + K_o(mr)I_1(mr_o)}{I_o(mr_i)K_1(mr_o) + K_o(mr_i)I_1(mr_o)}$$
(31)

$$(\eta_{fin})_{c} = \frac{2r_{i}}{m(r_{o}^{2} - r_{i}^{2})} - \frac{I_{1}(mr_{o})K_{1}(mr_{i}) - K_{1}(mr_{o})I_{1}(mr_{i})}{I_{1}(mr_{o})K_{o}(mr_{i}) + K_{1}(mr_{o})I_{o}(mr_{i})} (32)$$

To check the validity of the present numerical model, the numerical results for temperature distribution and fin efficiency for clean convective heat transfer from fins with different values of clean mL were compared with the corresponding analytical solutions. The comparisons for temperature distribution along the fin and fin efficiency are shown in Figs 5 and 6, respectively. It is obvious that the output of the numerical model is in good agreement with the analytical solutions for all values of fin parameter mL.



Fig. 5 Comparison between numerical and analytical solutions for fin temperature distribution for clean convective heat transfer.



Fig. 6 Comparison between numerical and analytical solutions for fin efficiency for clean convective heat transfer.

4.2 Effect of fouling on the performance of fins during nucleate boiling

The effect of fouling on the temperature distribution and the fin efficiency for nucleate pool boiling conditions are analyzed for different values of "Bi_f" starting from clean conditions (Bi_f = 0) up to a severe level of fouling (Bi_f = 4).

4.2.1 Temperature distribution

The temperature distribution in a clean fin and its variation due to fouling during nucleate pool boiling is shown in Fig. 7 for two different fin parameters $(mL)_c$ of 1 and 3. For clean and fouled conditions, increasing the fin parameter mL leads to decreasing temperature and greater temperature non-uniformity. This clearly underlines the effects of fin height, fin thickness and fin thermal conductivity on the performance of the finned tube during boiling.

As the deposit begins to grow on the fin sides and base, the base temperature of the fin increases, resulting in higher temperatures at the surface of the fins. As seen in Fig. 7, the rise in temperature of the fin is higher towards the tip for all configurations. Hence, the presence of fouling increases the fin temperature and also affects the temperature profile along the fins.



Fig. 7 Effect of fouling on fin temperature distribution during nucleate pool boiling.

4.2.2 Fin efficiency

Fig. 8 shows the effect of fouling on the efficiency of fins for different fin parameters. As a general trend, increasing the fin parameter "mL" results in a reduction of the fin efficiency. This efficiency trend as a function of "mL" is the reason for the limited use of medium- and high-finned tubes in boiling applications.

Under fouling conditions, to determine the time dependence of the efficiency ratio, the fouling rate, dBif/dt, must be known. As indicated in Fig. 8, the efficiency of the fins is increased for all levels of fouling and "mL". Referring to Eq. (5) which defines the fin parameter "mL", it is clear that the fouled fin parameter (mL)_f is always less than the clean one (mL)_c, from which it follows that the

fouled fin efficiency is greater than the clean fin efficiency. This is due to:

- increased temperature uniformity in the fin in the event of fouling;
- larger driving temperature difference between fin surface temperature and bulk temperature, and therefore increased contribution of fins to the total heat transfer.

The efficiency ratio of the fouled to the clean fin for different fin parameters as a function of non-dimensional Biot number " Bi_{f} " is shown in Fig. 9. The efficiency ratio is always higher than unity for all values of "mL".



Fig. 8 Effect of fouling on fin efficiency during nucleate pool boiling.



Fig. 9 Fouled to clean fin efficiency ratio versus nondimensional time for different clean fin parameters during nucleate pool boiling.

5. CONCLUSIONS

A one-dimensional numerical model has been developed to calculate the thermal performance of finned tubes in terms of the temperature profile of the fins and their thermal efficiency, for both convective and nucleate boiling conditions. For the case of convective heat transfer, the output of the numerical model is in close agreement with the analytical solutions for all values of fin parameter "mL".

The temperature distributions across the fin as well as the fin efficiency were calculated during nucleate boiling under both clean and fouling conditions. The results show that the presence of fouling increases the base temperature of the fins and the temperature gradient along the fins, consequently decreasing the boiling heat transfer coefficients.

The model also underlines an important feature of fins such as their increased efficiency when subjected to fouling. As explained numerically, the improved temperature uniformity in the fin in the event of fouling accounts for this effect.

NOMENCLATURE

- A surface area, m^2
- Bi Biot number
- I_n, K_n *nth* order modified Bessel functions of the first and second kind
- k thermal conductivity, W/mK
- mL fin parameter
- n, n' indexes in Equations 14 and 16
- q heat transfer rate, W
- r radius, m
- R_f fouling resistance, m²K/W
- s mean thickness, m
- t time, s
- T temperature, °C
- X normalized distance

Subscripts

- b base, bulk
- c clean, cross section
- f fouled
- fb fin base fin finned
- i inner
- max maximum
- o outer

Greek symbols

- α heat transfer coefficient, W/m²K
- θ normalized temperature
- η efficiency
- τ normalized time

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