

LINEAR PARAMETER-VARYING IDENTIFICATION OF A CROSS FLOW HEAT EXCHANGER FOR FOULING DETECTION: A PRELIMINARY STUDY

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ABSTRACT

In this article, the problem of building a mathematical model of a cross flow heat exchanger based on observed data from a numerical simulation system is studied. In order to explicitly take into account the dependence of the system dynamics on the hot and cold mass flow rates, a particular model structure, named linear parameter-varying (LPV) model, is considered. By using this structure, an output error identification problem is formulated. A parameter estimation scheme is introduced in which a cost function is minimized using a non-linear programming method. In this study, a finite volume digital model simulator is used to simulate and generate data. Using this simulator, a three step approach is carried out to get a model of a clean cross flow heat exchanger. The outputs of this clean model are then compared to the outputs of the simulator in which fouling is simulated by introducing a fouling factor inside the heat exchanger. This simulation shows that fouling detection can be easily performed.

INTRODUCTION

Fouling has been and is still a major concern in a great number of industrial processes such as refineries, dairy factories, district heating and so on. Apart from local measuring techniques such as proposed, *e.g.*, in (Bott, 2000) or (Ismail, et al., 2004), model-based techniques are more and more popular to monitor or to detect fouling. Although some interesting results can currently be found in the model-based fault detection literature (Lalot, et al., 2007), (Jonsson, et al., 2007), (Lalot and Mercère, 2008), new developments must be proposed to deal with non-linear grey-box models (Ljung, 1999). Indeed, most of the algorithms exploited until now can only be applied on linear time-invariant (LTI) systems, condition which may be restrictive in practice. In this communication, this problem is sorted out by introducing a particular model structure: a linear parameter-varying (LPV) model (Shamma, 1996). An LPV system is more precisely a finite dimensional linear system whose state-space or transfer function entries depend on time-varying parameters (named the scheduling variable) assumed to be measurable signals. The development of such models is mainly linked to control engineering where a control system must be

designed in order to guarantee the suitable closed loop operation of a given plant in many different operating conditions. A well known example of controller design technique using this basic idea is the gain scheduling approach (Shamma and Athans, 1992).

In the identification framework considered hereafter, the LPV structure allows the assessment of a non-linear system by deriving a parameter-dependent model on the basis of local experiments, *i.e.*, experiments in which the scheduling variable is held constant and only the control input is excited. Such a viewpoint has been considered in (Steinbuch et al., 2003), (van Helvoort et al., 2004), (Paijmans et al., 2008) for motion and robot control. Some recent developments concerning this identification procedure can also be found in (Lovera and Mercère, 2007). Hereafter, estimation of the parameters for each constant scheduling variable model is performed by using a particular output error (Trigeassou et al., 2003). Then, these local models are interpolated using a classic least squares algorithm to get an LPV model of the process. Thus, roughly speaking, the global behaviour of the system is embedded using a particular set of interpolated LTI models.

In order to validate this identification procedure on heat exchanger input-output (I/O) data, a mathematical model of a cross flow heat exchanger is used. Contrary to (Jonsson et al., 2007) where a commercial CFD tool is used, a particular numerical simulation model is employed. This simulator is similar to models given in (Kou and Yuan, 1997), (Mishra et al., 2008). The model can accurately simulate transient behaviour of different inputs and properties, which is necessary when mimicking real inflow situations with and without fouling.

The paper is organized as follows. In the second Section, the mathematical model of the cross flow heat exchanger is explained. Then, the third Section is dedicated to the identification procedure by stressing on the LPV model building phase. The experimental specifications and the simulation results are given in the fifth Section. The sixth Section concludes this communication.

SIMULATION MODEL

This section describes the simulation model used to generate data for outflow conditions in the heat exchanger. The simulator should represent the physics of the problem in an accurate way and, if this is fulfilled, it can be argued that the validity of the identification methods can be proven with simulated data representing the real world.

Mathematical formulation

The model is based on a mathematical representation of two fluids flowing perpendicular to each other, separated with a fixed wall. Various parameters can be adjusted in order to represent different sizes of cross flow heat exchangers, as seen in Figure 1.

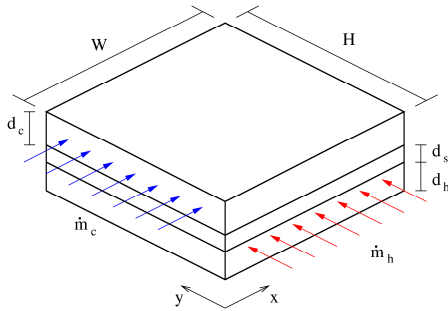


Fig. 1 A simple illustration of a cross flow heat exchanger.

The state of the heat exchanger at a given point in time is represented by three field variables $T_c(x, y)$, $T_h(x, y)$ and $T_s(x, y)$ representing respectively the temperatures of the cold fluid, the hot fluid and the wall (the subscript s denotes steel in this case). Three coupled partial differential equations describe the temperature fields, namely

$$\rho_c c_c d_c \frac{\partial T_c}{\partial t} + \frac{\dot{m}_c c_c}{H} \frac{\partial T_c}{\partial x} = U_c (T_s - T_c) \quad (1)$$

$$\rho_h c_h d_h \frac{\partial T_h}{\partial t} + \frac{\dot{m}_h c_h}{W} \frac{\partial T_h}{\partial y} = U_h (T_s - T_h) \quad (2)$$

$$\rho_s c_s d_s \frac{\partial T_s}{\partial t} = U_c (T_c - T_s) + U_h (T_h - T_s) \quad (3)$$

These equations describe advection (convection) without diffusion, but with source terms on the right hand side that represent heat flow from the hot fluid to the wall and from the wall to the cold fluid. It is furthermore assumed that the inflow is uniform for both fluids and that complete mixing takes place just before the fluids exit each passage. Note that the heat capacity of the wall $\rho_s c_s$ will delay the heat transfer between fluids in dynamic situation, which is the main reason for including the wall.

To summarize, the inflow temperature, mass flow rate and heat transfer coefficient U are generally assumed to be time dependent in the model while other parameters are constant. Thus, Eq. (1-3) represent a real heat exchanger if they are solved in a consistent and accurate manner.

Numerical scheme

Various numerical methods are available to deal with advective-diffusive problems, both finite element methods as, e.g., (Westerink et al., 1989) or and also finite difference methods, see, e.g., (Leonard, 1979). Comparisons between available methods, see, e.g., (Choi et al., 1995), indicate that the method presented in (Leonard, 1979) as well as an improved version in (Leonard, 1991) can be considered as the state of the art for such a problem. Therefore, the QUICKEST numerical scheme is applied for the advective part of the problem. The only boundary conditions in this case are the inflow temperatures, which require alternative treatment for finite difference points close to the inlets, as discussed in (Sousa, 2007).

The source terms on the right hand sides of Eq. (1-3) are approximated by a central difference method in time, while the advection terms are explicit according to the QUICKEST scheme. Therefore the time stepping method hereafter can be regarded as a semi-implicit method. The Courant number characterizes advection and is defined as

$$Co_c = \frac{\dot{m}_c}{\rho_c d_c H} \frac{\Delta t}{\Delta x} \quad (4)$$

$$Co_h = \frac{\dot{m}_h}{\rho_h d_h W} \frac{\Delta t}{\Delta y} \quad (5)$$

for the cold and hot side of the heat exchanger. Now let the superscript j denote the current time-step and $j+1$ a new one. Also, let $\hat{T}_c^j$ and $\hat{T}_h^j$ denote temperatures calculated with the QUICKEST scheme, which would result in a pure advective temperature at time-step $j+1$.

Then the coupled system becomes

$$\left[C + \frac{\Delta t}{2} A \right] \begin{bmatrix} T_c^{j+1} \\ T_s^{j+1} \\ T_h^{j+1} \end{bmatrix} = \left[C - \frac{\Delta t}{2} A \right] \begin{bmatrix} T_c^{j+1} \\ T_s^{j+1} \\ T_h^{j+1} \end{bmatrix} - C \begin{bmatrix} \hat{T}_c^j \\ 0 \\ \hat{T}_h^j \end{bmatrix} \quad (6)$$

where C and A are matrices respectively defined as

$$C = \begin{bmatrix} \rho_c c_c d_c & 0 & 0 \\ 0 & \rho_s c_s d_s & 0 \\ 0 & 0 & \rho_h c_h d_h \end{bmatrix} \quad (7)$$

$$A = \begin{bmatrix} U_c & -U_c & 0 \\ -U_c & U_c + U_h & -U_h \\ 0 & -U_h & U_h \end{bmatrix} \quad (8)$$

The result is a numerical scheme for updating the three fields in time, with an accurate high order advection part and a source part based on central differences in time. There is one stability requirement that must be fulfilled, which is that both Co_c and Co_h must be smaller than one.

IDENTIFICATION OF THE LPV MODEL

A cross flow heat exchanger is by construction a non-linear system. Then, a particular model structure must be

used to get an accurate model of the system behaviour. Because it is really difficult to find *a priori* non-linearities mathematical models of such a system, the procedure applied in this communication consists in

1. estimating several local LTI models, by applying classic estimation methods dedicated to LTI systems, on the basis of local experiments in which a particular variable, representing the current operating point, is held constant and the control input is (persistently) excited,
2. interpolating these local LTI models to get finally a single LPV model encompassing the global behaviour of the system in a compact manner.

By this way, the final model is able to reproduce efficiently the dynamical behaviour of the non-linear system without requiring complex algorithms for non-linear identification.

LPV model

The LPV model used hereafter has the following structure

$$T_{co}(p) = H_{cc}(p)e^{-\delta_c p}T_{ci}(p) + H_{ch}(p)T_{hi}(p) + G_{cc}(p)\dot{M}_c(p) + G_{ch}(p)\dot{M}_h(p) \quad (9)$$

$$T_{ho}(p) = H_{hc}(p)T_{ci}(p) + H_{hh}(p)e^{-\delta_h p}T_{hi}(p) + G_{hc}(p)\dot{M}_c(p) + G_{hh}(p)\dot{M}_h(p) \quad (10)$$

where p is the Laplace transform symbol, $T_{co}(p)$ and $T_{ho}(p)$ are respectively the Laplace transform of the cold and hot output temperatures, $T_{ci}(p)$ and $T_{hi}(p)$ are respectively the Laplace transform of the cold and hot input temperatures, $\dot{M}_c(p)$ and $\dot{M}_h(p)$ are respectively the Laplace transform of the cold and hot mass flow rates, $H_{\ell n}(p)$ is the transfer functions relating the output temperature ℓ and the input temperature n and $G_{\ell n}(p)$ is the transfer functions relating the output temperature ℓ and the input mass flow rate n .

It is important to notice that the T_{co} (resp. T_{ho}) is not directly linked to T_{ci} (resp. T_{hi}). Indeed, a delay δ_c (resp. δ_h) is required to take into account the transportation duration of the cold fluid (resp. hot fluid) in the cross heat exchanger. More particularly,

$$\delta_c = \frac{\rho_c d_c WH}{\dot{m}_c} \quad \text{and} \quad \delta_h = \frac{\rho_h d_h WH}{\dot{m}_h} \quad (11)$$

As it was said before, the parameters of the LPV model explicitly depend on a scheduling variable assumed to be measurable when the system is working. Here, the scheduling variable is the vector

$$\mathbf{\kappa} = \begin{bmatrix} \dot{m}_c \\ \dot{m}_h \end{bmatrix} \quad (12)$$

This choice is linked to the fact that these two signals are measured and controlled easily on real heat exchangers. Thus, the assumption that the scheduling variable is held constant during the local experiments can be satisfied.

Here, the LPV structure exclusively concerns the transfer functions $H_{\ell n}(p)$, $\ell, n \in \{c, h\}$. Under the assumption that each transfer function $H_{\ell n}(p)$ is second order, i.e.,

$$H_{\ell n}(p) = \frac{b_{\ell n}}{p^2 + a_{2\ell n}p + a_{1\ell n}} = \frac{K_{\ell n}}{1 + 2\frac{\zeta_{\ell n}}{\omega_{\ell n}}p + \frac{p^2}{\omega_{\ell n}^2}} \quad (13)$$

with $K_{\ell n}$ the static gain, $\zeta_{\ell n}$ the damping ratio and $\omega_{\ell n}$ the natural frequency of the model, the LPV structure is introduced by assuming that the parameters $b_{\ell n}$, $a_{1\ell n}$ and $a_{2\ell n}$ satisfy the generic relation

$$\begin{aligned} \gamma_{\ell n} &= (\beta_0 + \beta_1 \dot{m}_c + \beta_2 \dot{m}_c^2)(\lambda_0 + \lambda_1 \dot{m}_h + \lambda_2 \dot{m}_h^2) \\ &= \alpha_0 + \alpha_1 \dot{m}_h + \alpha_2 \dot{m}_h^2 + \alpha_3 \dot{m}_c + \alpha_4 \dot{m}_h \dot{m}_c \\ &\quad + \alpha_5 \dot{m}_h^2 \dot{m}_c + \alpha_6 \dot{m}_c^2 + \alpha_7 \dot{m}_h \dot{m}_c^2 + \alpha_8 \dot{m}_h^2 \dot{m}_c^2 \end{aligned} \quad (14)$$

where γ stands for b_1 , a_1 or a_2 . On the contrary, the parameters of the transfer functions $G_{\ell n}(p)$ are assumed to be invariant with respect to $\dot{m}_c$ and $\dot{m}_h$.

Output error LTI model identification of $H_{\ell n}(p)$, $\ell, n \in \{c, h\}$

The first step of the identification procedure consists in holding the mass flow rate signals constant and identifying the LTI transfer functions for each operating point $\mathbf{\kappa}$. Assuming that $\dot{m}_c$ and $\dot{m}_h$ are fixed, it is obvious that

- the delays δ_c and δ_h are constant,
- the coefficients $b_{1\ell n}$, $a_{1\ell n}$ and $a_{2\ell n}$ (or $K_{\ell n}$, $\zeta_{\ell n}$ and $\omega_{\ell n}$) are invariant,
- the influence of the terms $G_{\ell n}(p)\dot{M}_n(p)$, $\ell, n \in \{c, h\}$, on the dynamics of the system is null. Indeed, as soon as $\dot{m}_c$ and $\dot{m}_h$ are fixed, these elements are constant.

These observations have three interesting practical consequences:

- Firstly, the effects of the constant time-delays can be easily overcome by shifting the input data T_{ci} and T_{hi} with respect to the values of δ_c and δ_h after the I/O data acquisition.
- Similarly, the influence of $G_{\ell n}(p)\dot{M}_n(p)$, $\ell, n \in \{c, h\}$ can be cancelled during the data treatment phase when the means of the I/O data are removed.
- Finally, since the parameters of the transfer functions to assess are constant, classic LTI system identification methods can be used to identify the local models.

Considering all these remarks, the LTI models to identify satisfy

$$T_{co}(p) = H_{cc}(p)T_{cid}(p) + H_{ch}(p)T_{hi}(p) \quad (15)$$

$$T_{ho}(p) = H_{hc}(p)T_{ci}(p) + H_{hh}(p)T_{hid}(p) \quad (16)$$

where T_{cid} and T_{hid} are the input data T_{ci} and T_{hi} after time-delay treatment.

Many algorithms are now available to identify such LTI models (Ljung, 1999). They are mainly characterized by their estimation mechanism, their accuracy and their ability to converge to the desired solution. In this communication, models with output error (OE) structures and dedicated minimization algorithms are used. The main interest in using OE identification algorithms is to provide asymptotically unbiased parameter estimation (Ljung, 1999). Unfortunately, this property is achieved at the cost of the minimization of a non-linear quadratic criteria, which generally leads to a non-uniqueness of the optimum (Pronzato, 2001). This fundamental problem can be solved partially by initializing these algorithms with the help of a model identified using an error equation technique (Ljung, 1999). In this paper, this initial step is realized using the prediction error method (PEM) (Ljung, 1999). Combining the PEM and the OE identification algorithm, the convergence towards the global optimum of the minimized cost function is ensured with high probability.

In order to explain the basic idea of the OE algorithm and make the notations clearer, consider the generic single input single output (SISO) relation

$$Y(p) = \frac{N(p)}{D(p)}U(p) \quad (17)$$

with $(n > d)$

$$N(p) = n_0 + n_1 p + \dots + p^n \quad (18)$$

$$D(p) = d_0 + d_1 p + \dots + d_d p^d \quad (19)$$

and assume that M I/O data pairs $\{u_k \ y_k^*\}_{k \in [1, M]}$ are available where y^* is the noisy measurement of the system output y , i.e., $y^* = y + v$ where v is a zero-mean noise. Then, introduce

$$\theta = [n_0 \ \dots \ n_{n-1} \ d_0 \ \dots \ d_d]^T \quad (20)$$

the vector of the system's parameters to estimate and $\hat{\theta}$ an estimation of θ . Assuming that an initial vector $\hat{\theta}$ is available¹, the model output response due to an input u can be simulated easily. Noting by $\hat{y}$ this simulated output, the residuals can be constructed

$$\varepsilon_k = y_k^* - \hat{y}_k \quad (21)$$

and the quadratic criterion can be calculated

$$J = \sum_{k=1}^M \varepsilon_k^2 \quad (22)$$

The optimal value of θ is obtained by minimizing the quadratic cost function J . Since $\hat{y}$ is non-linear in the parameters $\hat{\theta}$, J has to be minimized iteratively via a non-linear programming algorithm. For this purpose, the Marquardt's algorithm (Marquardt, 1963) can be used. This algorithm estimates θ iteratively as follows

$$\theta_{(i+1)} = \theta_{(i)} - \left\{ J''_{\theta\theta} + \lambda I \right\}^{-1}_{\theta=\theta_{(i)}} J'_{\theta} \quad (23)$$

where (i) stands for the i th iteration, λ is the monitoring parameter and

$$J'_{\theta} = -2 \sum_{k=1}^M \varepsilon_k \sigma_k \quad \text{is the gradient} \quad (24)$$

$$J''_{\theta\theta} = -2 \sum_{k=1}^M \sigma_k \sigma_k^T \quad \text{is the hessian} \quad (25)$$

$$\sigma_k = \frac{\partial \hat{y}_k}{\partial \theta_{(i)}} \quad \text{is the output sensitivity function} \quad (26)$$

This algorithm ensures robust convergence, even with a bad initialization of θ . Fundamentally, this technique is based on the gradient and hessian calculation. These functions are dependant on the numerical integration of the sensitivity functions σ_k . The sensitivity functions can be efficiently integrated by simulating a set of state-space models (see (Lee and Poolla, 1999) for details about this smart numerical method).

Applying this algorithm to the I/O data related to Eq. (15)-(16), the invariant parameters $b_{\ell n}$, $a_{\ell n}$ and a_{2n} of the LTI transfer functions $H_{\ell n}(p)$, $\ell, n \in \{c, h\}$ can be estimated for each operating point $\mathbf{k}$.

Interpolation procedure

Assuming that q sets of four LTI transfer functions $\{H_{\ell n}(p)\}_{\ell, n \in \{c, h\}}$ (corresponding to q operating points) are available, the interpolation of the identified parameters can be examined. In fact, the same procedure is used for each parameter $\gamma_{\ell n}$, $\ell, n \in \{c, h\}$ (see Eq. (14)) and will be explained only for this generic notation. Assume that the local experiments have been carried out for constant $\dot{m}_c \in [\dot{m}_c^{(1)} \ \dots \ \dot{m}_c^{(q)}]$ and $\dot{m}_h \in [\dot{m}_h^{(1)} \ \dots \ \dot{m}_h^{(q)}]$. Then, it is easy to see that $\gamma_{\ell n}$ can be represented by a particular surface in 3 dimensions whose equation has to be calculated. Having access to q values of $\gamma_{\ell n}$ corresponding each to a couple $\{\dot{m}_c, \dot{m}_h\} \in [\dot{m}_c^{(1)} \ \dots \ \dot{m}_c^{(q)}] \times [\dot{m}_h^{(1)} \ \dots \ \dot{m}_h^{(q)}]$, the coefficients α_j of Eq. (14) can be easily estimated from the following least squares problem

$$\begin{bmatrix} \gamma_{\ell n}^{(1)} \\ \vdots \\ \gamma_{\ell n}^{(q)} \end{bmatrix} = [\alpha_0 \ \dots \ \alpha_8] \begin{bmatrix} 1 & \dots & 1 \\ \vdots & & \vdots \\ \dot{m}_h^{(1)} & & \dot{m}_h^{(q)} \\ \dot{m}_h^{(1)2} \dot{m}_c^{(1)2} & \dots & \dot{m}_h^{(q)2} \dot{m}_c^{(q)2} \end{bmatrix} \quad (27)$$

¹ This initial vector is obtained by using PEM.

Since all the data involved in this least squares problem are noise-free, the estimated parameters $\hat{\alpha}_j$ are consistent using a classic least squares algorithms (see, *e.g.*, (Ljung, 1999) for details about linear regression).

As soon as all the coefficients α_j for each parameter $\gamma_{\ell n}$, $\ell, n \in \{c, h\}$ are available, the LPV structure of the model can be built.

Output error LTI model identification of $G_{\ell n}(p)$, $\ell, n \in \{c, h\}$

The last step of the identification problem concerns the estimation of the parameters of the transfer functions $G_{\ell n}(p)$, $\ell, n \in \{c, h\}$, relating the cold and hot output temperatures and the cold and hot mass flow rates. To reach this goal, two ways can be suggested.

The first one, applied in this communication, assumes that the inputs T_{c_i} and T_{h_i} can be controlled and fixed constant and the mass flow rates $\dot{m}_c$ and $\dot{m}_h$ persistently excited. Then, as in the previous case, the effect of $H_{\ell n}(p)$, $\ell, n \in \{c, h\}$, on the dynamics of the system is null and can be easily removed during the data treatment phase. Hence, the system behaviour is governed by the following relations

$$T_{c_o}(p) = G_{cc}(p)\dot{M}_c(p) + G_{ch}(p)\dot{M}_h(p) \quad (28)$$

$$T_{h_o}(p) = G_{hc}(p)\dot{M}_c(p) + G_{hh}(p)\dot{M}_h(p) \quad (29)$$

Again, an OE algorithm can be used to estimate the parameters of the LTI transfer functions $G_{\ell n}(p)$, $\ell, n \in \{c, h\}$.

When the inputs T_{c_i} and T_{h_i} cannot be held as constant values, the parameters of $G_{\ell n}(p)$, $\ell, n \in \{c, h\}$ can be computed by following a three-step procedure

1. knowing $H_{\ell n}(p)$, $\ell, n \in \{c, h\}$ and the inputs T_{c_i} and T_{h_i} , simulate the corresponding temperatures outputs using Eq. (15)-(16),
2. remove these simulated outputs from the system outputs acquired during the global experiment,
3. estimate the LTI transfer functions $G_{\ell n}(p)$, $\ell, n \in \{c, h\}$ from these modified outputs and the mass flow rates inputs by using, *e.g.*, an OE algorithm.

RESULTS

The identification procedure described beforehand is now applied on I/O data simulated from the cross heat exchanger simulator described in the second Section.

Firstly, 49 local experiments; corresponding to 7×7 constant couples $\{\dot{m}_c, \dot{m}_h\} \in [0.7:0.1:1.3] \times [0.7:0.1:1.3]$ are carried out. For each local experiment, (persistently excited) cold and hot temperatures inputs are applied.

These inputs are built using splines, centred around 20°C for the cold side and 38°C for the hot side, with frequency changes chosen with respect to the transient dynamic of the system (see Fig. 2). In order to take account the measurement disturbances classically encountered in practice, two zero-mean white Gaussian noises, with noise-signal ratios equal to 20dB, are added on the simulated outputs T_{c_o} and T_{h_o} .

The first step is the data treatment. After removing the means of each local experiment I/O data set, the time-delay is cancelled by shifting the inputs with respect to the values of δ_c and δ_h computable for constant $\dot{m}_c$ and $\dot{m}_h$ with Eq. (11).

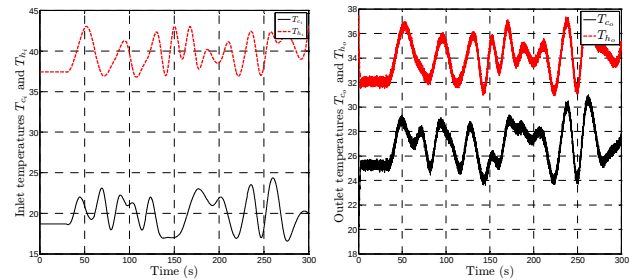


Fig. 2 Example of I/O data. $\dot{m}_c = 0.9$ kg/s and $\dot{m}_h = 1.1$ kg/s.

Then, the OE algorithm, initialized with the PEM algorithm, is applied on these treated I/O data to get the parameters of the 196 local LTI transfer functions $H_{\ell n}(p)$, $\ell, n \in \{c, h\}$. These models are validated on a second set of noise-free I/O data (see Fig. 3). The fit given on Figure is computed as follows:

$$\text{FIT} = 100 \times \left(\frac{1 - \|y - \hat{y}\|}{\|y - \text{mean}(y)\|} \right) \quad (30)$$

For each local LTI transfer functions $H_{\ell n}(p)$, $\ell, n \in \{c, h\}$, the fit is between 80 and 98%.

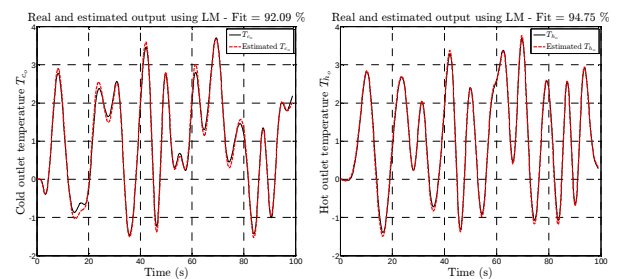


Fig. 3 Validation of the OE model. $\dot{m}_c = 0.9$ kg/s and $\dot{m}_h = 1.1$ kg/s.

Having access to the parameters of these 196 LTI models, the interpolation step can be performed. To justify the use of second order polynomials (see Eq. (13)), the evolution of the static gain K_p and the damping ratio ζ

with respect to $\dot{m}_c$ and $\dot{m}_h$ are plotted in Figure 4. It is quite obvious that quadratic functions are sufficient to fit these curves. To compute the coefficients of these functions, Eq. (27), associated with a classic least squares algorithm, is used. Figure 5 illustrates the efficiency of this approach on a particular parameter.

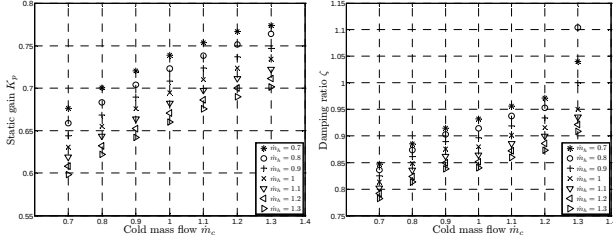


Fig. 4 Evolution of K_p and ζ with respect to $\{\dot{m}_c, \dot{m}_h\}$.

The last step consists in validating the ensuing LPV model. For that, three situations are analysed. Firstly, the same conditions as in the previous validation step are used, *i.e.*, a new set of noise-free I/O data corresponding to constant mass flow rates values are probed. Again, the fit is greater than 80% (see Fig. 6). Then, this LPV model is tested with $\dot{m}_c = 0.5$ kg/s and $\dot{m}_h = 1.5$ kg/s, *i.e.*, values out of the range used during the identification phase. The fit values given in Figure 7 indicate that the LPV model performs quite well.

Finally, specifications corresponding to $\dot{m}_c = 0.9$ kg/s and $\dot{m}_h = 0.8$ kg/s for $t < 50$, $\dot{m}_h = 1.2$ kg/s for $t > 50$ are applied. In this case (see Fig. 8), the fit is highly lower after the $\dot{m}_h$ change. Thus, a direct relation between the outputs T_{c0} and T_{h0} and the mass flow rates seems to be necessary. This is done by identifying the transfer functions $G_{\ell n}(p)$, $\ell, n \in \{c, h\}$.

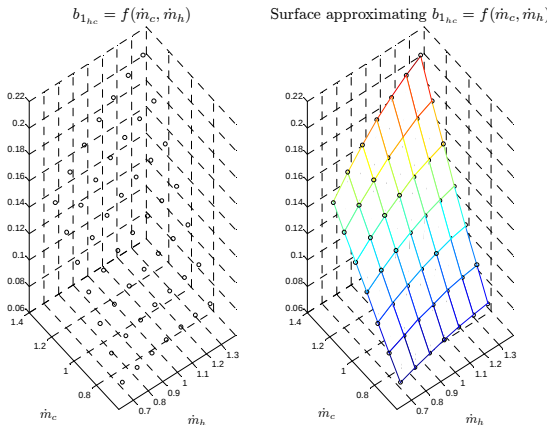


Fig. 5 Repartition of the estimated coefficients $b_{1_{ch}}$ with respect to $\{\dot{m}_c, \dot{m}_h\}$ and surface approximating the relation between these signals.

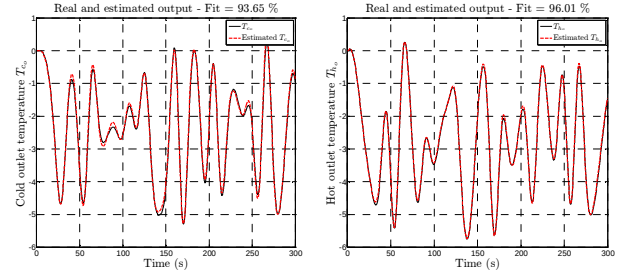


Fig. 6 Validation of the LPV model. $\dot{m}_c = 0.9$ kg/s and $\dot{m}_h = 1.1$ kg/s.

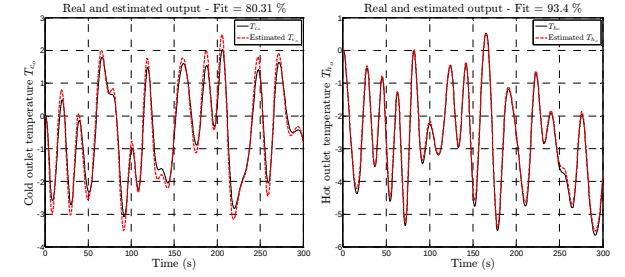


Fig. 7 Validation of the LPV model. $\dot{m}_c = 0.5$ kg/s and $\dot{m}_h = 1.5$ kg/s.

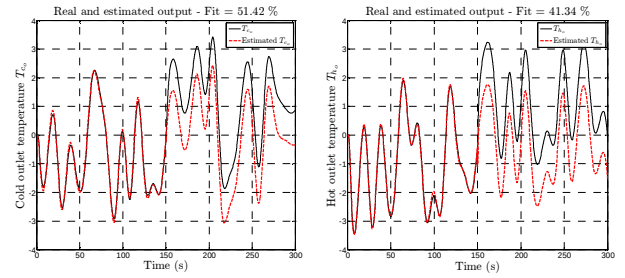


Fig. 8 Validation of the LPV model. $\dot{m}_c = 0.9$ kg/s and $\dot{m}_h = 0.8$ kg/s for $t < 50$, $\dot{m}_h = 1.2$ kg/s for $t > 50$.

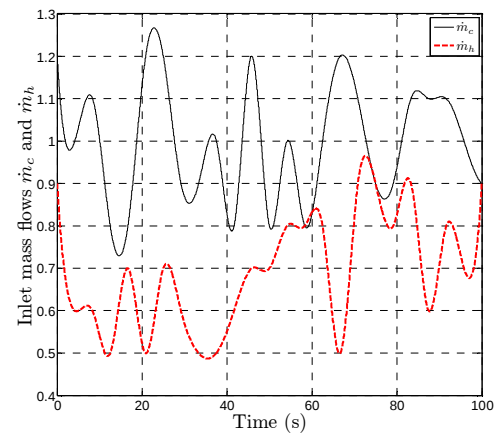


Fig. 9 I/O data for the validation of the global model.

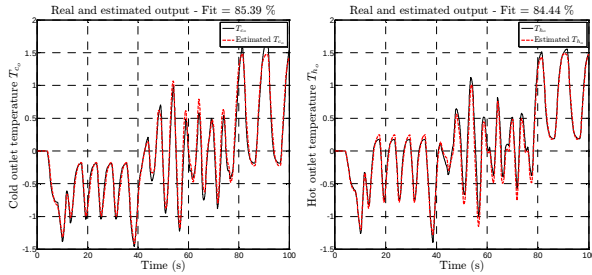


Fig. 10 Validation of the global model.

Adding these LTI transfer functions to the LPV structure identified previously, the global model is able to reproduce the dynamical behaviour of the simulator for time-varying T_{ci} , T_{hi} , $\dot{m}_c$ and $\dot{m}_h$, as shown in Figure 10.

The last experimental result concerns the fouling detection. The approach considered in this communication consists in comparing the outputs of a reference model, identified from clean I/O data, with the outputs of the system under fouling influence. To simulate fouling, the thermal conductivity of the hot side h_h is continuously decreased (see Fig. 11). More precisely, in order to be sure that the evolution of the chosen detection signal is due to fouling and not, e.g., transient behaviour, the data base is composed of two sets: the first one corresponds to a clean period, the second one to the progressively fouled heat exchanger.

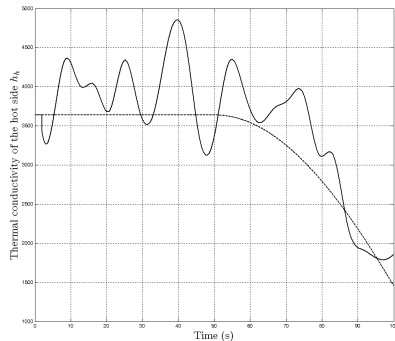


Fig. 11 Evolution of the hot side thermal conductivity.

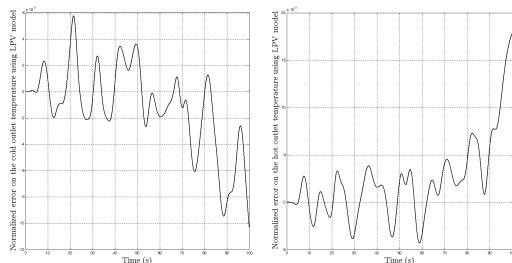


Fig. 12 Evolution of the hot normalized error for the cold and hot sides.

As shown in Figure 12, the normalized errors on the hot and cold temperatures evolve as soon as the hot thermal conductivity decreases (instant 50). Using a particular statistical test, such as, e.g., the CUSUM test

(see, e.g., (Basseville and Nikiforov, 1993)), fouling can be easily detected.

Remark: Notice that the most difficult part of this study is the determination of a reliable but easy-tuning model of the system. As soon as an accurate model is available, the problem of fouling detection is only related to the choice of the reliable signal(s) leading to fouling detection (here the system outputs), then the statistical test to apply in order to warn the user as soon as fouling is occurring. This last problem can be solved by applying the test the user is used.

CONCLUSIONS

In this communication, the problem of identifying a cross flow heat exchanger has been considered. More precisely, a particular model structure, named linear parameter-varying model, has been used. By this way, the identified model has been able to efficiently reproduce the dynamical behaviour of the non-linear cross flow heat exchanger without requiring complex algorithms for non-linear identification. For that, a local experiments procedure has been carried out. Based on I/O generated from a reliable cross heat exchanger simulator, the identification procedure proposed in this paper has shown interesting performances which lead to believe that it can be surely extended to data acquired from a real cross flow heat exchanger. This problem will be addressed in the futures works.

NOMENCLATURE

- c specific heat, J/kg K
- Co Courant number, dimensionless
- d passage thickness, m
- H heat exchanger height, m
- $\dot{m}$ mass flow rate, kg/s
- p laplace transform symbol, dimensionless
- t time, s
- T temperature, K
- U overall heat transfer coefficient, W/m² K
- W heat exchanger width, m
- x spatial dimension, m
- y spatial dimension, m
- δ time-delay, s
- Δ difference operator, dimensionless
- ρ density, kg/m³

Subscript

- c cold
- h hot
- i input
- o output
- s steel

Superscript

- j timestep

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